SUMMER MEETING AT CHICAGO
CONFERENCE REPORTS

CONFERENCE ON ALGEBRA AND TWENTY-THIRD COLLOQUIUM

The Conference on Algebra at the 1941 Summer Meeting of the Society in Chicago was held in three sessions. The first dealt with abstract algebra, especially with lattice theory, and the lectures were given by Professors John von Neumann and Garrett Birkhoff. The session was preceded by the first Colloquium Lecture of Professor Oystein Ore on the allied topic of Mathematical Relations and Structures. The second session was concerned with topics in linear algebra and the theory of matrices and the speakers were Professors Nathan Jacobson and N. H. McCoy. General arithmetic notions came to the fore in the third and final session, with talks by Professors J. F. Ritt and Oscar Zariski, and Dr. O. F. G. Schilling. There was considerable discussion, both organized and spontaneous, after all the talks.

1. *Abstract algebra and lattices*

A number of different algebraic systems can be subsumed under the notion of a lattice-ordered group (or an *l-group*). Professor Birkhoff's talk dealt with the structural properties of such groups. By definition, an *l*-group is a group which is also a lattice and which has the "homogeneity" property ($x \leqq y$ implies $a+x \leqq a+y$ and $x+a \leqq y+a$). Examples include the additive groups of ordered fields, partially ordered function spaces (Kantorovitch and others), and the lattice of all ideals in an integral domain (Clifford, Lorenzen, Krull).

After discussing the elementary properties of *l*-groups and the variety of possible postulate systems, Birkhoff turned to their structure theory. The study of homomorphisms of *l*-groups leads naturally to a concept of an *l*-ideal. The lattice of all *l*-ideals is distributive. A central result asserts that every *l*-group with a chain condition is either directly decomposable, or has exactly one maximal proper *l*-ideal.

Many of the results known for abelian *l*-groups hold without serious change for non-abelian groups. However, any *l*-group in which there is a chain condition for the elements is necessarily abelian. There is a number of unsolved questions: Does there exist a non-abelian *l*-group which is complete, in the sense that every bounded set of elements has a least upper bound? Is there a theory of *l*-rings? Are there simple (non-abelian) *l*-groups which are not simply ordered?

von Neumann began by a comparison of classical and quantum

mechanics. Any two operators (observables) of classical mechanics can be added *and* multiplied together. But in quantum mechanics, the observables, being hermitian operators, can be added but not multiplied together, unless they are permutable. Hence they form a commutative group only, and not a ring. This group is partially ordered if the positive semi-definite operators are called "positive." Unfortunately, this ordering does *not* have the lattice property, again unlike classical mechanics. By assuming the lattice property of classical mechanics, but not the ring property, we get a physical background for the study of complete lattice-ordered abelian groups which are not rings.

These have very special structure. Consider the complete modular lattice M of all subgroups of such a group G. In M, the l-ideals of G form a distributive sublattice D, closed in M with respect to unrestricted unions and intersections. The closed l-ideals form a complemented sublattice B of D (hence a Boolean algebra), which is closed in D under unrestricted intersections, but only under finite unions.

Furthermore, using B and the representation theory of Boolean algebras, due to Stone, by bicompact totally disconnected topological spaces, one can obtain a representation theory for G by real functions on such spaces. In this representation, addition corresponds to addition; however, a function is to be called positive if the set on which it assumes negative values is nowhere dense. At any point, a function may assume either all integral or all real values.

2. *Linear algebras and matrices*

Many of the properties of matrices ordinarily proved only for matrices with elements in a field can actually be stated and proved, in appropriate form, for matrices whose elements lie in more general rings. McCoy's conference lecture was concerned with results of this character, especially for noncommutative rings. One of the accomplishments of the E. H. Moore General Analysis was a definition of a determinant for a hermitian matrix whose elements are quaternions. The same technique can be applied to define the determinants of suitably restricted matrices over much more general noncommutative rings. However, the proofs of some of the appropriate properties of these determinants require new devices.

The determinant can be used to define a characteristic polynomial of a matrix A and to prove the analogue of the Cayley-Hamilton theorem that every matrix satisfies its characteristic equation. The minimal polynomial of an ordinary matrix is replaced by a suitable minimal *ideal*, composed of all polynomials of which the matrix is a

root. In terms of the elements of the adjoint matrix one can determine a condition of the usual form that a polynomial belong to this minimal ideal. The theorems of Phillips and Ostrowski about equations satisfied by several matrices can also be generalized to matrices with elements in a ring.

Jacobson's talk dealt with Lie Algebras. Let A be an associative algebra of characteristic p. It is well known that the elements of A form a Lie algebra if one defines $[ab]=ab-ba$. Jacobson pointed out that in addition, the following identities hold

$$(I) \qquad (a+b)^p = a^p + b^p + s(a, b),$$

$$(II) \qquad [\cdots[[ab]b]\cdots b] = [ab^p],$$

provided $s(a, b)$ is a suitably defined, but complicated, Lie polynomial in a and b. He defined a *restricted* Lie algebra of characteristic p as a Lie algebra with operation $a\to a^p$ which satisfies (I)–(II) and $(\alpha a)^p = \alpha^p a^p$.

He mentioned, as other examples of restricted Lie algebras, the skew-symmetric elements of A under any anti-automorphism of A, and the "derivations" of *any* algebra (associative or not). A derivation is a linear operator D which satisfies, for all x, y, $(xy)D=(xD)y+x(yD)$, and corresponds, over the real field, to an infinitesimal automorphism.

Conversely, Jacobson showed that any restricted Lie algebra L could be realized by a Lie subalgebra of a suitably constructed associative algebra U_L. (The corresponding result for ordinary Lie algebras was known.) Sharper results were given. The subalgebra consists precisely of those elements of U_L which are skew under a certain *involution* of L. Moreover, if L has a finite basis, then so does U_L; hence L has only a finite number of inequivalent irreducible representations by matrices. If, in addition, every element of L is nilpotent, then U_L is a nilpotent associative algebra.

Finally, Jacobson showed that every derivation of U_L induced a "restricted" derivation of L, in the sense that $x^pD=[\cdots[xD, x]\cdots x]$, *and conversely*, every restricted derivation of L was induced by some derivation of U_L. These results have impelled him to the conclusion that restricted Lie algebras of characteristic p are closer analogues of Lie algebras of characteristic ∞ than ordinary (unrestricted) Lie algebras of characteristic p.

3. *General arithmetic*

The theory of polynomial ideals is concerned essentially with the manifold of solutions of polynomial equations in many variables,

but there has been developed a parallel theory for formal differential equations and the corresponding manifolds of solutions. Ritt's lecture summarized some of the recent results in this theory, emphasizing the respects in which this theory diverges from the polynomial case.

Starting with a form F which is a polynomial in the unknowns $y_1, \cdots, y_n$ and their derivatives, with coefficients in some (differential) field K, there is a corresponding manifold M, which consists of all the zeros $a_1, \cdots, a_n$ of F lying in some extension of the coefficient field K. There is a unique decomposition of such a manifold into "irreducible" manifolds, which corresponds to the decomposition of a "perfect" differential ideal into an intersection of prime ideals. However, there is no exact analogue of the ordinary algebraic theorem which gives a decomposition of arbitrary ideals into primary components. The dimension of an irreducible manifold can be appropriately defined, but the dimension of an intersection does not behave properly. Among the irreducible components of the manifold of a form, one is essential, in the sense that it does not occur in the manifolds of other forms. Ritt quoted theorems giving conditions that a given manifold be essential for a form in one variable. These conditions, originally found by analytic methods, have been derived in part by purely algebraic arguments, in a recent paper by H. Levi. The discussion after the lecture concerned some new basis theorems for systems of forms over fields of prime characteristic.

The famous and difficult problem of the reduction of the singularities of an algebraic manifold by birational transformations was the subject of Zariski's conference lecture. An r-dimensional manifold is given by a field of algebraic functions of r independent variables. Any n elements $x_1, \cdots, x_n$ which generate this field determine a projective model of the manifold in the n-dimensional projective space with the (nonhomogeneous) coordinates $x_1, \cdots, x_n$. The problem consists in finding some model of the manifold in which there will be no singular points.

Zariski has published an algebraic solution of this problem, for two-dimensional manifolds. His talk discussed the generalization of this attack to higher dimensions. Two steps are necessary. The first is that of local uniformization of the "places" of the manifold. A place is essentially a valuation of the associated field of algebraic functions. The uniformization theorem asserts that there is a projective model of the manifold on which the "center" of the valuation is a simple point. This step has been carried out for r-dimensional manifolds.

The second step is that of getting this uniformization to hold simultaneously for all places. Zariski discussed the procedures available

here, which consist essentially in a suitable application of locally quadratic transformations. He announced that his methods now were sufficiently powerful to solve the problem of reducing the singularities of a three-dimensional manifold, in the case where the field of coefficients is the complex number field.

Schilling's talk dealt with a generalized Hilbert theory for fields with a valuation. The original Hilbert theory concerns the successive stages in the decomposition of a rational prime p into prime ideals in a normal algebraic number field. Alternatively, one may start, not with the field of rational numbers, but with the associated p-adic number field. In the general case the prime p is replaced by a valuation, and one requires that the field F be relatively complete with respect to this valuation (this assumption is more appropriate than the stronger requirement of topological completeness). Any normal extension of such a field F may be obtained in two uniquely determined steps. The first step is an "inertial" extension, in which the valuation is unramified (that is, the corresponding prime ideal is not decomposed). The second step is a completely ramified extension, in which the residue class field is not extended. There is a corresponding decomposition of the Galois group into two parts. The whole theory can be carried through for infinite extensions, and the resulting Galois groups can be explicitly characterized. In case the residue class field is finite, and the order of the group is a power of a prime, the reverse problem can be solved: one obtains necessary and sufficient conditions that a given group can be realized as the Galois group of a normal extension of the given base field. The whole theory is analogous to a generalized "Kummer theory" for function fields.

4. *The Colloquium Lectures*

The object of these lectures is to make a general study of the properties of mathematical relations as they occur in the various mathematical theories. One may illustrate such relations by various examples, for instance, the relations: one element contains another, a point is middle point between two given ones, a set is the closure of another set. One finds that a more general type of relations may be considered as correspondences $A \rightarrow R_A$ between sets, where the set R_A corresponding to a set A may depend not only upon the elements in A, but also upon a certain grouping and ordering of the elements in A, as, for instance, in the case of noncommutative or non-associative multiplication. If an element a in a set S belongs to the subset R_A it is convenient to write this as a relation aRA. Such relations may be combined in various ways and certain normal forms of relations may

be introduced. Particularly interesting is the problem of finding automorphisms or endomorphisms of a relation R, namely those one-to-one or many-to-one correspondences α such that any relation aRA is taken into another valid relation $a^{\alpha}RA^{\alpha}$. The determination of all automorphisms is connected with the problem of finding all correspondences commuting with a given correspondence. It should also be mentioned that these problems lead to a study of the so-called monomial groups and to problems connected with a general Galois theory, extending the ordinary Galois theory of equations.

Of particular importance are the so-called binary relations aRb between two elements a and b. As examples, one may think of a greater than b, a equivalent to b, a orthogonal to b, a divides b, and so on. The theory of such binary relations may be conceived of in many ways. The first systematic theory of such binary relations was given by Peirce and by Schroeder. In their formulation one can say that the theory appears as a theory of matrices over a Boolean ring. Another method of representing the binary relations consists in conceiving the relation as a graph, joining two vertices a and b by an edge from a to b whenever aRb. The symmetric relations are those for which aRb imply bRa. They correspond to the symmetric matrices or to graphs in which the edges are considered undirected.

When the relations are considered to be matrices one can also unite their theory with vector spaces over special rings. For the relations one can define various operations: sum and intersection, multiplication and dual multiplication. Thus it becomes possible to introduce a ring and ideal theory of binary relations and certain parts of the ordinary theory of linear algebras carry over to relations. In other ways the theory of relations differs considerably from ordinary matrix theory. Also here one is interested in the automorphisms and endomorphisms of the relations. It turns out that the automorphisms correspond to the permutation matrices which commute with the relation matrix, and this property may be used for the determination of automorphisms.

Of particular interest in the theory of relations are the transitive relations for which

$$aRb,\ bRc \text{ implies } aRc.$$

These relations may be characterized in the ring of all relations in various ways, for instance, by the property $R^2 \subset R$ or also as certain units or as certain sum relations. They are closely associated with the idempotents of the rings of relations. Any transitive relation can be broken up in an equivalence relation and a partial order relation. For

the equivalence relations a number of important algebraic problems can be solved completely, and the interest is therefore centered upon the partially ordered sets. These sets have gained an increasingly important position in mathematical theories, and a number of useful properties of these sets are known. Only certain aspects of these theories are discussed in detail in these lectures.

Among these partially ordered sets the lattices or structures are particularly important. In these systems there exists a unique minimal element $a \cup b$ containing any two elements a and b, and a unique maximal element $a \cap b$ contained in both. These elements $a \cup b$ and $a \cap b$ may also be considered to be the result of two algebraic operations with certain simple properties. Recent investigations have shown that many mathematical theories may be formulated in terms of such structures, and the systematic use of these concepts gives a unification and a simplification of the various theories. As special instances one may mention the set theory of Stone (Harvard), projective geometry by Garrett Birkhoff (Harvard) and von Neumann (Institute for Advanced Study), euclidean and non-euclidean geometry by Menger (Notre Dame), algebraic theories by the lecturer. Among the many other recent contributions to lattice theory, one can mention papers by Dilworth (Yale) and Ward (California Institute of Technology) on multiplication in such systems, by MacLane (Harvard) on certain types of lattices applicable to chains in algebraic systems, by Glivenko on metric lattices, by Jacotin-Dubreils on equivalence relations. In these lectures only certain specific problems, mainly of an algebraic nature, can be discussed with some completeness.

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CONFERENCE ON THE THEORY OF INTEGRATION

The sessions of this conference were held concurrently with those of the Conference on Algebra. The following paragraphs give a relatively inadequate summary of the lectures, which provided a very illuminating introduction to the researches of the various speakers, especially in their relation to the historical development of the theory of integration.

The first session was devoted to theories of integration in abstract spaces, and both speakers were concerned with extreme generalizations of the notion of the upper and lower integrals of Darboux. Professor Bochner, in his lecture entitled *Integration and partial ordering*,

discussed the problem of extending the domain of a positive and finitely additive operator M on V to Y, when V, V' and Y are partially ordered vector spaces with $V \subset V'$, and Y is also assumed to be a complete lattice. For an arbitrary element ϕ of V' the extension $M\phi$ satisfies the inequalities

$$M_*\phi \equiv \sup_{f \leqq \phi} Mf \leqq M\phi \leqq \inf_{f \geqq \phi} Mf \equiv M^*\phi,$$

where f ranges over V. The largest subset of V' on which the extension of M is uniquely determined is the set $\overline{V}$ on which $M^*\phi = M_*\phi$. The space $\overline{V}$ is complete in the Riemann-Darboux sense, that is, if θ in V' and ϕ_n and ψ_n in $\overline{V}$ are such that

$$\phi_n \leqq \phi_{n+1} \leqq \theta \leqq \psi_{n+1} \leqq \psi_n, \quad \sup M\phi_n = \inf M\psi_n,$$

then θ belongs to $\overline{V}$. In certain special cases, when the elements of V' are point functions, and the space $\overline{V}$ contains sufficiently many step-functions for purposes of approximating to $M\phi$ for arbitrary ϕ in $\overline{V}$, the operation $M\phi$ may appropriately be considered as an integral $\int \phi(x) d\mu(x)$. Bochner showed how the theory may be applied to a generalization of the classical moment problem, that is, to secure a representation of a completely monotone function $f(x)$ with values in Y in the form

$$f(x) = \int_0^\infty e^{-xt} d\mu(t).$$

Among other applications mentioned was the spectral resolution of a bounded hermitian operator in Hilbert space. The discussion of Bochner's paper was initiated by Dr. R. S. Phillips.

Professor Price, in his lecture entitled *Integration and convex operators*, began with an historical discussion of the use of the upper and lower sums of Darboux, leading up to the generalization of this notion in the "integral range" of Garrett Birkhoff. Later developments in this direction by Price himself required the use of a generalized convex operator. In the present discussion the speaker showed that the notion of a convex operator is entirely unnecessary. In contrast to the approach of Bochner outlined above, Price assumes a "measure function" T_σ given in advance, in terms of which an integral operator $M(f) = \int f dT$ is constructed. No partial ordering of the domain or the range of M is used. The integral range is constructed as follows. Let V be a vector space, S a class of elements σ, and f a function of σ whose values are subsets of V. Let T_σ, for each σ in S, be an additive transformation of V into V, let Δ denote a finite subset $(\sigma_1, \cdots, \sigma_n)$ of S, and let D be a class of such subsets Δ which constitutes a directed

set in the sense that it is partially ordered and has the composition property. The integral range of f associated with Δ is defined by the formula

$$I(f, \Delta) = \sum_{\Delta' \geqq \Delta} [T_{\sigma_1'} f(\sigma_1') + \cdots + T_{\sigma_n'} f(\sigma_n')],$$

where $\Delta' = (\sigma_1', \cdots, \sigma_n')$, $\sum$ denotes the logical sum, and $+$ denotes the addition of complexes. This integral range obviously has the following properties: (1) $\Delta_2 > \Delta_1$ implies $I(f, \Delta_2) \subseteq I(f, \Delta_1)$; (2) $I(f_1 + f_2, \Delta) \subseteq I(f_1, \Delta) + I(f_2, \Delta)$. No topological properties of the space V enter into the definition of $I(f, \Delta)$. However, the method of defining a class of integrable functions and an integral $\int f dT$ associated with the class of integral ranges $[I(f, \Delta)]$ depends on the topology in V. If V is a Banach space, we may say that f is integrable in case the lower bound of the diameters of its integral ranges is zero, and $\int f dT$ is then the unique element common to the closures of the integral ranges of f. The integral has the usual properties when suitable restrictions on the basis are made. The speaker indicated the solution of this topological problem of integration also in the case when the space V is a space (L) of Fréchet. Professor Nelson Dunford initiated the discussion of this paper.

Professor Hildebrandt began his lecture on *The general integral of E. H. Moore* by pointing out the three-fold character of Moore's investigations into theories of integrals. Moore first interested himself in improper integrals of the Harnack type and then in his two forms of general analysis. The central feature of both forms is the introduction of a completely unrestricted range $\mathfrak{P}$ on which is defined the class $\mathfrak{M}$ of complex- (or real-) valued functions to be discussed. The first form of general analysis, a group of postulational theories, was an attempt to correlate certain theories of linear equations, including the Fredholm theory. In one formulation of this analysis two classes $\mathfrak{M}_1$ and $\mathfrak{M}_2$ of the type mentioned above are considered, and the integral is a bilinear, continuous operator, continuity being defined with respect to a relative uniform convergence topology. In the case $\mathfrak{M}_1 = \mathfrak{M}_2$, the operator J may be taken to be properly positive and hermitian and is thus the forerunner of the basis of a general Hilbert space.

To illustrate how the second form was developed Hildebrandt discussed the Schmidt theory of the system of equations

$$\sum_{i=1}^{\infty} x_i a_{ij} = y_j, \qquad i = 1, 2, \cdots,$$

and then indicated the extension to the case when the variables i and j have general ranges. For functions μ_1, μ_2 of a class of "modular functions" an integral $J\mu_1\mu_2$ is defined which is a bilinear, properly positive and hermitian operator. If $J\mu_1\mu_2$ is taken to be the inner product (μ_1, μ_2), the space of modular functions is seen to be a generalized Hilbert space (with no restriction on the dimension number). Among the important features of Moore's general analysis from the point of view of present day mathematics are his use of a completely unrestricted domain $\mathfrak{P}$ for his modular functions, his conception of an integral as a bilinear operator, and his exploitation of the Moore-Smith limit, which has led to the modern use of directed sets. In discussing the paper Professor Barnard, among other things, developed the modular space determined by functions $f(x-y)$ of a type discussed in the previous lecture by Bochner.

Professor Jeffery began his discussion of *Non-absolutely convergent integrals* with a brief historical summary of the early work of Denjoy and Perron, recalling that it was not until 1924 that Alexandroff and Looman showed the equivalence of the Perron integral with the special Denjoy integral. The problem of formulating an integral of Perron type equivalent to the general Denjoy integral was solved by Ridder in 1933. The speaker gave Ridder's definition and an outline of the equivalence proof. He next discussed the approximately continuous integral of Burkill and also his "Cesàro-Perron" integral of the first order, of all positive integral orders, and finally of all real orders, with a brief description of the application of Burkill's work to Fourier series. For a list of Burkill's papers on these subjects the reader is referred to volume 11 of the Journal of the London Mathematical Society.

In conclusion Jeffery discussed an approach to the problem of non-absolutely convergent integrals which was originated by himself and M. S. Macphail. A function $F(e)$ of the measurable sets e on the interval (a, b) is absolutely-additive relative to the closed set E if it is completely-additive over E and if for the intervals (α_i, β_i) complementary to E, $\sum F(\alpha_i,\beta_i) = F[\sum(\alpha_i,\beta_i)]$. $F(e)$ is generalized absolutely-additive on (a, b) if this interval can be covered by a denumerable sequence of closed sets relative to each of which $F(e)$ is absolutely-additive. Let $f(x)$ be measurable and let $s_n(x)$ be a sequence of summable functions tending to $f(x)$ such that $\int_a^x s_n\,dx$ tends to a limiting function $F(x)$. There is thus associated with $F(x)$ a function $F(e) = \lim \int_e s_n dx$, provided this limit exists. If it turns out that $F(e)$ is generalized absolutely-additive on (a, b), then $F(x)$ is the non-absolutely convergent integral of $f(x)$. This integral is equivalent to the

general Denjoy integral. The approach lends itself to a simple and brief treatment which quickly brings out all the main facts about non-absolutely convergent integrals. It is also easily adapted to integrals of the Stieltjes type, and to a formulation of a theory of non-absolutely convergent integrals in abstract space. The discussion of Jeffery's lecture was led by Professor McShane.

In introducing his talk on *Measure in statistical mechanics* Professor Wiener considered a measure-conserving mapping T of the interval (0, 1) into itself and remarked that the set Σ of points P for which $a_i \leqq T^iP < b_i$ $(i=0, \pm 1, \pm 2, \cdots, \pm n)$ is measurable. As a consequence, then, of the ergodic theorem, it follows that $\lim_{N=\infty}$ (Number of values of k in (1, N) for which T^kP is in $\Sigma)/N = m\Sigma$ and for almost all values of P *this will be true for all such sets* Σ *simultaneously*. A sequence $\{x_k\}$ $(k=0, \pm 1, \pm 2, \cdots)$ is called by Wiener a *time series*, and it is termed *regular* if the number of values of k in $(-N, +N)$ for which $a_i \leqq x_{k-i} < b_i$ $(i=0, \pm 1, \pm 2, \cdots, \pm n)$, when divided by $2N$, tends to a limit for every selection of the a_i, b_i. It was indicated above how the time series T^iP $(i=0, \pm 1, \cdots)$ is regular. Next Wiener showed how every regular time series generates a measure of time series together with a transformation under which this measure is invariant, or in other words establishes a situation under which ergodic theory may be applied. It may be shown that almost all time series are regular and generate the same measure among time series. Thus with respect to this measure, translation in time is metrically transitive. It can be shown that any measure-preserving transformation may be broken up into metrically transitive components. One of the greatest needs of ergodic theory is the technique of producing measure-preserving transformations with desired spectrum characteristics. It is generally desired that these transformations be mutually transitive. One of the easiest ways of producing such transformations is the explicit formation of regular time series. Wiener's theory of Brownian motion was in essence nothing but a proof that almost all Brownian motions generate the continuous analogue of a regular time series. Another interesting application of time series is in the theory of almost periodic functions. Wiener next gave a particularly interesting example due to himself and Kurt Mahler to show the existence of time series with non-absolutely continuous spectra. It is known that the metrically transitive transformation generated by this example is not even mixing in the weak sense, that is, the distribution of $x_k, \cdots, x_{k+j}$ is not even on the average independent of that of $x_{k+l}, \cdots, x_{k+l+j}$, where $l > j$. On the other hand, let $\{p_i\}$ be an increasing sequence of primes. Represent a number N in the form

$A_0+\sum_1^N A_k p_1 \cdots p_k$ $(A_k<p_{k+1})$ and let $x_{\pm n}=\prod_0^N \omega_k^{A_k}(n>0)$, where ω_k is a primitive p_{k+1}th root of unity. Then the time series x_n will be regular and the translation operator will be metrically transitive with respect to the measure thus generated. It will in addition be weakly mixing but is not strongly mixing. Wiener concluded his address by conjecturing that a slight generalization of such methods may be used to derive strongly mixing transformations which will be such that the spectrum (in the sense of his generalized harmonic analysis) of some sequence $f(T^N P)$ will be non-absolutely continuous. This would indicate that not all mixing transformations are isomorphic. The discussion was initiated by Professor von Neumann.

The talk on *Product integrals* by Professor Garrett Birkhoff began with a brief discussion of the solution, in terms of a product integral, of a system of ordinary linear differential equations, as developed by Volterra and Schlesinger. Here the matrix product replaces the ordinary sum in the definition of the integral. Birkhoff has generalized this notion, defining a product integral for functions whose values are in a complete normed vector ring with a unit. Thus, let $V(t)$ be defined on the interval (a, b), and let Π be a partition $(\Delta_1, \Delta_2, \cdots, \Delta_n)$ of this interval. Then the Riemann product integral is defined by the equation

$$\overbrace{\int_a^b} V(t)dt = \lim_{\Pi}[V(t_1)\Delta_1 \circ V(t_2)\Delta_2 \circ \cdots \circ V(t_n)\Delta_n],$$

where t_k is in Δ_k, provided the limit exists in the Moore-Smith sense. The ring operation $\circ$ must be distinguished from the "scalar multiplication" by Δ_k. It should be noted that when the operation $\circ$ is interpreted as ordinary addition, the process of product integration just defined reduces to integration in the ordinary sense. Birkhoff indicated that while the notion of product integral may clearly be applied to linear operators on any Banach space, it may also be so interpreted as to apply to non-linear operators. In this case two norms are needed in the space of operators. The distributive law which holds in any ring may be replaced by approximate distributivity (that is, differentiability) near the unity element. In the finite-dimensional case, this permits the integrands to have values in any Lie group; the matrix-valued integrands of Volterra and Schlesinger are thus included as a very special case. In the infinite-dimensional case, the values can be in any "analytical group" (see Birkhoff's paper in Journal of Mathematics and Physics, vol. 16 (1937), pp. 104–132). An extension of the notion of product integral indicated above to one of

Lebesgue type was discussed by the speaker. Certain properties have been secured, but many other problems remain to be elucidated. The connection between product integrals and the fundamental group in the simplest case of a surface was exhibited by Professor Rainich, whose remarks were communicated in a letter read to the conference.

The lecture on *Applications of the theory of integration to the study of linear operators* by Professor Dunford stated some of the unsolved problems as well as the known facts about one type of representation of linear maps between Lebesgue spaces, and also gave an outline of the method of Dunford and Pettis for attacking these problems. If $\psi = U\phi$ is a bounded linear map of $L^p(T)$ onto $L^q(S)$, where S and T are finite intervals of the real axis, it has a representation of the form

$$\psi(s) = \frac{d}{ds}\int_T H(s,t)\phi(t)dt. \tag{1}$$

In case $p=1$ and the map U is weakly compact (that is, U takes the unit sphere into a set whose closure is weakly compact), it has the simpler representation

$$\psi(s) = \int_T K(s,t)\phi(t)dt. \tag{2}$$

However, in case $p=q=1$ there are maps of the form (2) which are not weakly compact, and a topological characterization of the integral operator (2) between L spaces is unknown. One of the results stated by Dunford in this connection asserts that (2) (with $p=1\leqq q$) takes weakly compact sets into compact sets. This shows that the product of two weakly compact operators in L is a compact operator and makes it clear why certain aspects of the Fredholm theory hold for linear equations involving a weakly compact operator in L.

A fundamental class of problems concerning the operators (1) and (2) involves the characterizing in terms of the kernels H and K of those operators U which are bounded, weakly compact, compact, lattice convergence-preserving, and so on. These problems have for the most part been solved in case $p=1\leqq q$ even in the case of abstract S and T, but in all other cases very little is known. One method of investigation which has proved fruitful in the case $p=1$ consists of the following three steps: first, represent the operator U as an abstract integral $U\phi=\int_T\phi(t)x(t)dt$; second, find the relations between the set $J=[(1/|e|)\int_e x(t)dt]$ and the set of values $x(T)$ of the kernel; third, express the abstract integral as a numerical integral, that is, $\int_e x(t)dt = \int_e K(s,t)dt$ where $x(t)=K(\cdot,t)$. For bounded operators U on L to

X the first problem is solvable with a Gelfand integral in case X is the adjoint of a separable space and with a Bochner integral (for arbitrary X) in case U is weakly compact. The principal result under the second problem is that for measurable Pettis integrable functions there is a null set $E_0 \subset T$ such that $x(T-E_0)$ and J have the same closed convex hull. The representation desired in the third step always holds for Bochner integrable functions and at least for those Gelfand integrable functions arising in the representation of linear operations on $L(T)$ to $L^q(S)$. Clearly, it is the second step above which allows us to state the topological properties of the operator (at least those which are invariant under the operation of convex closure) in terms of the kernel. A partial list of those papers treating the problems mentioned above is contained in a paper by Dunford and Pettis appearing in the Transactions of this Society, vol. 47 (1940), pp. 323–392. Dunford's paper was discussed by Professor J. W. Calkin.

H. H. GOLDSTINE
L. M. GRAVES

THE ANNUAL MEETING OF THE SOCIETY

The forty-eighth Annual Meeting of the American Mathematical Society was held at Lehigh University, Bethlehem, Pennsylvania, from Monday, December 29 to Wednesday, December 31, 1941, in conjunction with meetings of the Mathematical Association of America, the Association for Symbolic Logic, and the National Council of Teachers of Mathematics.

There were two general and seven sectional sessions of the Society at which three addresses and seventy-nine research papers (forty-four in person) were given. All sessions were held in Packard Laboratory.

Arrangements for the meetings were made by a committee of which Dean Tomlinson Fort was chairman and Professors Everett Pitcher, G. E. Raynor, and W. M. Smith were the local members. Through the generosity of Lehigh University, rooms were available in the University dormitories.

The attendance was about three hundred fifty including the following two hundred fifty-six members of the Society:

C. R. Adams, R. B. Adams, R. P. Agnew, Leon Alaoglu, C. B. Allendoerfer, H. A. Arnold, H. P. Atkins, Frank Ayres, H. M. Bacon, N. H. Ball, I. A. Barnett, F. S. Beale, A. A. Bennett, Stefan Bergman, A. H. Black, W. A. Blankinship, Salomon Bochner, H. F. Bohnenblust, J. W. Bower, C. B. Boyer, J. W. Bradshaw, C. C. Bramble, R. W. Brink, H. W. Brinkmann, F. L. Brooks, R. H. Bruck, N. R. Bryan. S. S. Cairns, W. D. Cairns, P. A. Caris, M. E. Carlen, I. S. Carroll, W. F. Cheney, Claude Chevalley, D. E. Christie, Alonzo Church, Randolph Church, R. F. Clippinger I. S. Cohen, L. W. Cohen, Nancy Cole, J. B. Coleman, Esther Comegys, R. H. Cook, Richard Courant, A. P. Cowgill, H. B. Curry, E. H. Cutler, D. R. Davis, J. E. Davis, L. J. Deck, F. F. Decker, C. H. Denbow, R. P. Dilworth, L. L. Dines, Jesse Douglas, T. L. Downs, Arnold Dresden, J. C. Durand, J. J. Eachus, Samuel Eilenberg, W. E. Ferguson, F. A. Ficken, N. J. Fine, W. W. Flexner, M. M. Flood, L. R. Ford, Tomlinson Fort, R. M. Foster, J. S. Frame, R. E. Gaskell, H. M. Gehman, H. P. Geiringer, F. J. Gerst, B. P. Gill, A. M. Gleason, J. S. Gold, Michael Goldberg, H. H. Goldstine, Cornelius Gouwens, H. S. Grant, C. H. Graves, L. J. Green, Lewis Greenwald, F. L. Griffin, Theodore Hailperin, D. W. Hall, P. R. Halmos, H. H. Hartzler, G. E. Hay, Alan Hazeltine, E. R. Hedrick, A. E. Heins, M. H. Heins, Edward Helly, Erik Hemmingsen, Coleman Herpel, J. G. Herriot, H. C. Hicks, E. H. C. Hildebrandt, D. L. Holl, T. R. Hollcroft, Witold Hurewicz, W. A. Hurwitz, E. D. Jenkins, Fritz John, Evan Johnson, H. A. Jordan, Mark Kac, G. K. Kalisch, William Karush, J. L. Kelley, A. J. Kempner, R. B. Kershner, S. C. Kleene, R. B. Kleinschmidt, J. R. Kline, P. A. Knedler, T. L. Koehler, H. L. Krall, W. C. Krathwohl, H. N. Laden, K. W. Lamson, O. E. Lancaster, R. E. Langer, V. V. Latshaw, Solomon Lefschetz, Joseph Lehner, Norman Levinson, F. W. Light, M. I. Logsdon, E. N. Lorenz, C. I. Lubin, R. R. R. Luckey, J. J. McCarthy, N. H. McCoy, J. C. C. McKinsey, E. J. McShane, C. C. MacDuffee, G. W. Mackey, Saunders MacLane, H. M. MacNeille, H. F. MacNeish, P. T. Maker, V. S. Mallory, W. T. Martin, A. E. Meder, H. L. Meyer, E. R. C.

Miles, F. H. Miller, H. C. Miller, Deane Montgomery, Richard Morris, Marston Morse, E. J. Moulton, F. D. Murnaghan, C. A. Nelson, C. O. Oakley, Oystein Ore, E. G. Olds, F. W. Owens, H. B. Owens, J. C. Oxtoby, S. T. Parker, B. C. Patterson, G. W. Patterson, E. K. Paxton, E. W. Pehrson, J. W. Peters, C. R. Phelps, H. B. Phillips, R. S. Phillips, Everett Pitcher, J. C. Polley, Willy Prager, G. B. Price, A. L. Putnam, H. A. Rademacher, Tibor Radó, G. Y. Rainich, J. F. Randolph, C. H. Rawlins, G. E. Raynor, Maxwell Reade, O. H. Rechard, Haim Reingold, Moses Richardson, R. G. D. Richardson, R. F. Rinehart, E. K. Ritter, M. S. Robertson, R. E. Root, J. T. Rorer, Barkley Rosser, S. G. Roth, Hans Samelson, S. T. Sanders, L. J. Savage, S. A. Schelkunoff, I. J. Schoenberg, Abraham Schwartz, C. H. W. Sedgewick, I. E. Segal, Abraham Seidenberg, R. W. Shephard, Seymour Sherman, C. A. Shook, M. E. Sinclair, C. H. Sisam, L. L. Smail, D. M. Smiley, M. F. Smiley, P. A. Smith, T. L. Smith, W. M. Smith, Virgil Snyder, I. S. Sokolnikoff, R. H. Sorgenfrey, V. E. Spencer, G. W. Starcher, E. P. Starke, C. N. Stokes, R. W. Stokes, R. R. Stoll, A. H. Stone, Alvin Sugar, J. L. Synge, Otto Szász, J. D. Tamarkin, Alfred Tarski, R. M. Thrall, J. I. Tracey, W. R. Transue, A. W. Tucker, J. W. Tukey, J. L. Vanderslice, Oswald Veblen, R. W. Wagner, T. C. G. Wagner, R. J. Walker, P. R. Wallace, Henry Wallman, J. L. Walsh, André Weil, Alexander Weinstein, B. A. Welch, M. E. Wells, G. W. Whitehead, A. L. Whiteman E. A. Whitman, P. M. Whitman, Norbert Wiener, Audrey Wishard, W. D. Wray, Oscar Zariski.

The meetings of the Society opened Monday afternoon with two sections, Analysis and Algebra, at which Professor Tibor Radó and Dr. R. M. Thrall presided. Tuesday morning there were three sections, Analysis, Topology and Geometry, and Applied Mathematics, with Professors Norbert Wiener, L. W. Cohen, and L. R. Ford presiding.

Tuesday afternoon was devoted to a Symposium on Applied Mathematics, Professor F. D. Murnaghan presiding. Professor L. V. Bewley spoke on *The mathematical theory of traveling waves* and Professor I. S. Sokolnikoff on *Some new methods of solution of two-dimensional problems in elasticity*. The discussion leaders were Professors Alan Hazeltine, Ernst Weber, D. L. Holl, and J. L. Synge.

The annual business meeting and election of officers was held on Wednesday morning, President Marston Morse presiding. This was followed by the award of the Frank Nelson Cole Prize in the Theory of Numbers to Professor Claude Chevalley for his paper *La théorie du corps de classes*, published in Annals of Mathematics, (2), vol. 41 (1940), pp. 394–418. Professor Oscar Zariski then gave an address on *Normal varieties and birational correspondences*.

Wednesday afternoon there were two sections, Mathematical Logic, a joint session with the Association for Symbolic Logic, at which Professor A. A. Bennett presided; and Analysis and Algebra, at which Dr. Jesse Douglas presided.

Sessions of the Mathematical Association of America were held on Thursday morning and afternoon, those of the Association for Sym-

bolic Logic on Wednesday afternoon, and those of the National Council of Teachers of Mathematics on Wednesday and Thursday. There was a Pi Mu Epsilon luncheon on Thursday.

On Tuesday afternoon, the mathematicians and guests were entertained at a tea by the ladies of the Department of Mathematics of Lehigh University.

The attendance at the annual dinner held in the Hotel Bethlehem was two hundred sixty-two. The toastmaster, Professor W. H. Smith, introduced President C. C. Williams of Lehigh University who gave a brief address of welcome. This was followed by "In a Persian Garden" based on the Rubaiyat of Omar Khayyam sung by a quartet with piano accompaniment. Professor Virgil Snyder then read resolutions, prepared by a special committee appointed by the Council, in appreciation of the excellent services of Professor R. C. Archibald as Librarian of the Society for twenty-one years. These resolutions were adopted unanimously by a rising vote. They had been beautifully embossed on parchment for presentation to Professor Archibald. Professor J. R. Kline, Secretary of the Society, spoke of the work of the Society, its role in the present emergency and of plans for the future.

The program continued with a pleasing and well executed mathematical skit written by Dean Tomlinson Fort and presented by a cast of nineteen characters, revealing how Confusion has been dethroned by Mathematics. Following a cello solo by Professor K. W. Lamson, Professor I. S. Sokolnikoff presented resolutions expressing the appreciation of the mathematicians and guests to the President of Lehigh University, the members of the local committee and all who had assisted them for the excellent arrangements, pleasing entertainment, and cordial hospitality. At midnight, 1942 was welcomed with songs and New Year greetings.

Since the meeting, the local committee has reported that after all expenses of the meeting were paid, a small sum remained which was donated to the Red Cross.

At the meeting of the Board of Trustees at 6:00 P.M., December 29, 1941, in Lamberton Hall of Lehigh University, there was no quorum present. An adjourned meeting, therefore, was held on January 2, 1942, at 1:00 P.M. in the Faculty Club of Columbia University, New York City. There were two meetings of the Council, one being a joint session with the Board of Governors of the Mathematical Association of America. The joint session was held at 6:00 P.M. and the regular meeting of the Council at 8:20 P.M. on December 30, 1941 in Lamberton Hall.

At the joint meeting of the Council and Board of Governors, Presi-

dent Marston Morse presented the report of a subcommittee of the War Preparedness Committee (T. Y. Thomas, Chairman) which has been studying the problem of the Supply and Demand for Mathematicians in those institutions which confer the doctorate in mathematics. This study showed that in these institutions during the past year there has been a decrease of 23% in the number of graduate students and an increase of about 2% in the number of teachers of mathematics. It was reported also that there were indications of an increased demand for mathematical instruction of undergraduates. On the basis of this report, the Board of Governors and Council voted that steps should be taken with the proper authorities to have mathematicians employed as teachers in accredited colleges and universities and certain qualified graduate students recommended for individual occupational deferment.

The Secretary announced the election of the following thirty-nine persons to membership in the Society:

Mr. Gaynor Jefferson Adams, State Highway Department, Linden, Ala.;
Mr. Neal Russell Amundson, University of Minnesota;
Mr. Charles Cornelius Andersen, Jr., Chicago, Ill.;
Mr. Henry John Barten, Baltimore, Md.;
Mr. John Keppler Baumgart, Cumberland College, Williamsburg, Ky.;
Dr. Stefan Bergman, Brown University;
Mr. Robert George Blake, Hernando High School, Brooksville, Fla.;
Professor Harold E. Bowie, American International College, Springfield, Mass.;
Professor Mary L. Brady, College of Mount St. Vincent, New York, N. Y.;
Mr. Joseph Francis Brennan, Pacific Gas and Electric Company, San Francicso, Calif.;
Professor Myrtle Cyrena Brown, North Texas State Teachers College, Denton, Tex.;
Professor Irving Wingate Burr, Purdue University;
Mr. Edward Caleb Coker, Jr., Clemson College, Clemson, S.C.;
Mr. Douglas Richard Crosby, Royal Canadian Air Force, Rivers, Manitoba, Canada;
Dr. John A. Daum, Agricultural and Mechanical College of Texas;
Professor James E. Davis, Central Y.M.C.A. College, Chicago, Ill.;
Mr. James Riley Ellis, Booker T. Washington High School, Tulsa, Okla.;
Mr. Sidney Fernbach, Frankford Arsenal, Philadelphia, Pa.;
Mr. Marvin George Harrison, Curtiss Propeller, Caldwell, N.J.;
Mr. William R. Harvey, Bell Telephone Laboratories, New York, N.Y.;
Professor John Henry Hett, Manhattan College;
Mr. Franklin Ferguson Hopper, New York Public Library;
Mr. Walbert C. Kalinowski, St. John's University, Collegeville, Minn.;
Professor John Joseph McCarthy, St. John's University, Brooklyn, N.Y.;
Professor Russle Hindman MacCullough, Defiance College, Defiance, Ohio;
Mr. Joshua Matz, Yeshiva College, New York, N.Y.;
Dr. Harlan C. Miller, Winthrop College, Rock Hill, S.C.;
Mr. Hugh Jordan Miser, Ohio State University;

Professor Ernest Nagel, Department of Philosophy, Columbia University;
Mr. George R. Rich, Tennessee Valley Authority, Knoxville, Tenn.;
Sister Mary De Pazzi Rochford, Briar Cliff College, Sioux City, Iowa;
Professor Paul Anthony Samuelson, Massachusetts Institute of Technology;
Mr. Harry J. Sternberg, Stuyvesant High School, Brooklyn, N.Y.;
Miss Helen Farnam Story, St. Petersburg Junior College, St. Petersburg, Fla.;
Sister Mary Placide Thomas, Mt. Saint Agnes Junior College, Baltimore, Md.;
Dr. Peter Thullen, Departamento Actuarial del Instituto N. de Previsiòn, Quito, Ecuador;
Mr. Marvin Lewis Vest, West Virginia University;
Mr. Thomas Charles Gordon Wagner, University of Maryland;
Dr. Alexander Weinstein, University of Toronto.

It was reported that the following had been elected as nominees on the Institutional Memberships of the institutions indicated:

Brown University: Mr. Marshall Evans Munroe, Professor Willy Prager.
Bryn Mawr College: Miss Josephine Margaret Mitchell.
University of California at Los Angeles: Messrs. Paul Arnold Clement and Bill C. Moore.
University of Chicago: Mr. Roy Dubisch, Miss Anne Louise Lewis, Mr. J. Ernest Wilkins, Jr.
College of the City of New York: Mr. Harvey Cohn.
Columbia University: Miss Louise Comer, Mr. Leonard Gillman, Professor Jacques Hadamard, Messrs. Meyer Karlin and Ernst Gabor Straus.
Cornell University: Messrs. W. H. Durfee and R. R. R. Luckey.
Duke University: Mr. Bruce Elwyn Meserve.
Equitable Life Insurance Company of Iowa: Mr. Alton O. Groth.
Harvard University: Messrs. Edwin Hewitt, E. N. Lorenz, and A. L. Putnam.
University of Illinois: Mr. J. C. Bell, Miss Frances E. Ewing, Messrs. W. A. Ferguson, H. F. Gingerich, and N. M. Ginsberg, Miss Corinne Hattan, Mr. F. F. Helton, Misses Janie C. Lapsley and Naomi M. Livesay, Messrs. G. S. Mapes, E. B. Shanks, M. D. Springer, O. P. Staderman, and J. C. Stewart.
Indiana University: Miss Barbara Howe.
Institute for Advanced Study: Dr. Felix Adler, Professors Subrahmanyan Chandrasekhar and Kenneth Stewart Cole, Drs. Hans Samelson and Arthur Harold Stone.
Iowa State College: Mr. Ralph H. Tripp.
The State University of Iowa: Messrs. Louis Garfin and Edwin Halfar.
The Johns Hopkins University: Messrs. Richard Bellman, Seymore Abraham Fenichel, and Leslie G. Peck.
University of Kentucky: Mr. Paul W. Derthick.
Lehigh University: Mr. John Alden Pond.
Massachusetts Institute of Technology: Messrs. B. G. Farley, W. S. Loud, and O. G. Owens.
University of Michigan: Messrs. Wade Ellis, Albert A. Grau, and Jack Irwin Northam.
University of Minnesota: Messrs. William S. H. Crawford and William D. Munro.
Northwestern University: Mr. Herbert J. Greenberg.
University of Pennsylvania: Mr. Erik Hemmingsen.

Stanford University: Mr. Paul Charles Rosenbloom.
Swarthmore College: Professor André Weil.
Syracuse University: Mr. Robert C. Simpson, Jr.
University of Toronto: Mr. Wallace Charles Gordon Fraser.
University of Virginia: Messrs. Richard R. Bernard, William Aubrey Blankinship, and Mariano Garcia, Jr.
University of Washington: Mr. Arthur Henry Jerbert, Miss Dorothy Jeanne Morrow.
Wellesley College: Miss Alberta Schuettler.
Wesleyan University: Mr. Myron Edward White.
Yale University: Miss Martina Elizabeth Doyle, Mr. David McCray Merriell, Miss Mary Kent Peabody.

The following appointments by President Marston Morse were reported: as representative of the Society at the inauguration of Henry Elisha Allen as President of Keuka College (Keuka Park, N.Y.) on November 7, 1941, Professor W. B. Carver; as tellers for the election at the 1941 Annual Meeting, Professors E. R. Lorch and L. L. Smail; as new members of the Committees to Select Hour Speakers: for Annual and Summer Meetings, Professor M. H. Stone (committee now consists of Professors J. R. Kline, chairman, M. H. Ingraham, and M. H. Stone); for Eastern Sectional Meetings, Professor Salomon Bochner (committee now consists of Professors T. R. Hollcroft, chairman, C. R. Adams, and Salomon Bochner); for Western Sectional Meetings, Professor T. H. Hildebrandt (committee now consists of Professors W. L. Ayres, chairman, T. H. Hildebrandt, and W. J. Trjitzinsky); for Far Western Sectional Meetings, Professor G. C. Evans (committee now consists of Dean T. M. Putnam, chairman, Professors G. C. Evans, and T. Y. Thomas); as representative of the Society on the American Year Book for a period of three years, 1942–1944, Professor J. F. Ritt; as a new member of the Committee on Places of Meetings for a period of three years, 1942–1944, Professor P. R. Rider.

The Secretary reported that the ordinary membership in the Society is now 2,453, including 228 nominees of institutional members and 76 life members. There are also 87 institutional members. The total attendance of members at all meetings in 1941 was 1,574; the number of papers read was 474; the number of invited addresses was 38; the number of members attending at least one meeting was 949.

At the annual election which closed on December 31, and at which 492 votes were cast, the following officers were elected:

Vice Presidents, Professors C. C. MacDuffee and J. D. Tamarkin.

Associate Secretaries, Professors W. L. Ayres and M. H. Ingraham, Dean T. M. Putnam.

Librarian, Professor Arnold Dresden.

Members of the Editorial Committee of the Bulletin, Professor A. A. Albert, Dean Tomlinson Fort.

Member of the Editorial Committee of the Transactions, Professor Oscar Zariski.

Member of the Editorial Committee of the Colloquium Publications, Professor M. H. Stone.

Members of the Editorial Committee of Mathematical Reviews, Professors O. E. Neugebauer, J. D. Tamarkin, and Oswald Veblen.

Members-at-large of the Council, Professors G. A. Bliss, Nelson Dunford, W. T. Martin, Gabor Szegö, and S. S. Wilks.

The Council adopted the following resolutions concerning the work of Professor R. C. Archibald as Librarian of the Society:

The members of the American Mathematical Society take this opportunity to express their appreciation of the services of Raymond Clare Archibald on the occasion of his retirement from the post of its librarian.

During the twenty-one years in which he held this office he has greatly increased the size and usefulness of the library until it has become an important source of reference to all mathematicians. His conduct of this work has been marked by wholehearted devotion, extraordinary ability, and laudable economy.

The Society presents this memorial, attested by the signatures of its officers with assurances of the gratitude and affection of its members, with the hope that the Society may continue to receive the benefit of his advice and of his scholarly work for many years to come.

In an appendix to this report are excerpts from the Report of the Treasurer for the fiscal year 1941 as verified by the Auditors. A copy of the complete report will be sent, on request, to any member of the Society. The Board of Trustees adopted a budget for 1942 showing authorized expenditures of $49,050. This includes an appropriation of $1,000 for Mathematical Reviews from the Marion Reilly Fund.

The Librarian reported that the Library of the Society now contains 9,983 volumes of which 7,159 are bound volumes. Twenty-one years ago when Professor Archibald assumed the librarianship the corresponding numbers were 5,862 and 4,360. There are now over 150 periodicals on our exchange list corresponding to the earlier 123.

The American Journal of Mathematics, which is published jointly by The Johns Hopkins University and the Society, and to which the Society contributes an annual subvention of $2,500, printed 888 pages during 1941.

It was reported that a temporary editorial board consisting of Professors J. L. Walsh (chairman), A. A. Albert, and F. D. Murnaghan had been appointed in connection with the new series of books entitled *Mathematical Surveys*.

It was announced that copies of the recent report of Professor W. L. Hart (Chairman, Subcommittee on Education for Service) entitled *Mathematical Education for Defense* had been distributed in October to approximately 3,700 superintendents of schools throughout the country.

Times and places of meetings during 1942 were set as follows: Fall Far Western Meeting at the University of California at Los Angeles on November 28 and the 1942 Annual Meeting in New York City on December 28–30.

Certain invitations to give hour addresses were announced: Dr. R. P. Boas and Professor George Pólya for the April, 1942, meeting in New York City; Professor Salomon Bochner for the October, 1942, meeting in New York City.

Professor A. D. Michal was appointed Acting Associate Secretary for the far west during the period of Dean Putnam's illness.

Two books were accepted for publication in the Colloquium Series: *Algebraic Topology* by Professor Solomon Lefschetz and *Analytic Topology* by Professor G. T. Whyburn.

The Secretary announced that the Board of Trustees, upon recommendation of the Council, had adopted temporary regulations concerning the dues of those who are serving as enlisted men in the armed forces of the United States or Canada. The following provisions are included: (1) enlisted men who were members in good standing at the time of their induction may have their dues fixed at $1.00 per year during the period of their enlistment; (2) if they were subscribers to the Transactions or Mathematical Reviews at the time of their induction, they may continue to subscribe at the rate of $1.00 per year for the Transactions and $1.00 per year for Mathematical Reviews during the period of their enlistment. It is hoped that these men may thus be enabled to continue their contacts with mathematics while in the service. Full details of these regulations were mailed to members of the Society on January 1.

The Secretary announced that February 15 had been set as the final date for the receipt of orders for microfilm reading machines in connection with three-year subscriptions to Mathematical Reviews.

The Executive Committee of Mathematical Reviews reported that the subscription list of that journal, as of December 1, 1941, was 1,400.

Dean G. D. Birkhoff was elected Chairman of the Emergency Committee for the International Congress of Mathematicians, to replace Professor W. C. Graustein, deceased. It was announced that the Rockefeller Foundation had extended its appropriation for the International Congress to December 31, 1946.

President Marston Morse appointed as the Committee on Nomination of Officers and Members of the Council for 1943, Professors F. D. Murnaghan (chairman), A. A. Albert, H. A. Rademacher, W. M. Whyburn, and R. L. Wilder.

Titles and cross references to the abstracts of papers read at the regular sessions follow below. The papers were read as follows: Papers 1–5 in the section for Analysis on Monday afternoon; papers 6–12 in the section for Algebra on Monday afternoon; papers 13–18 in the section for Analysis on Tuesday morning; papers 19–25 in the section for Topology and Geometry on Tuesday morning; papers 26–33 in the section for Applied Mathematics on Tuesday morning; papers 34–39 in the section for Mathematical Logic on Wednesday afternoon; papers 40–44 in the section for Analysis and Algebra on Wednesday afternoon; and papers 45–79, whose abstract numbers are followed by the letter *t*, were read by title. Dr. Mickle was introduced by Professor Tibor Radó, Dr. Bergman by Professor T. R. Hollcroft, Dr. Miller by Professor R. L. Moore, Dr. Theilheimer by Professor R. E. von Mises, Professor Prager by Dean R. G. D. Richardson, Dr. Weinstein by Professor J. L. Synge. Paper 3 was presented by Professor W. T. Martin, paper 4 by Dr. G. E. Reves, paper 7 by Professor Smiley, paper 8 by Dr. Dilworth, paper 11 by Professor Barnett, paper 12 by Professor MacLane, paper 18 by Dr. Reade, paper 21 by Professor Weil, paper 41 by Professor MacLane, paper 43 by Dr. Phillips, and paper 44 by Dr. Thrall.

1. E. J. Mickle: *Associated double integral variation problems.* (Abstract 48-1-44.)

2. P. T. Maker: *The Cauchy theorem for functions on closed sets.* (Abstracts 48-1-43.)

3. Salomon Bochner and W. T. Martin: *A class of removable singularities in several complex variables.* (Abstract 48-1-32).

4. G. E. Reves and Otto Szász: *Some theorems on double trigonometric series.* (Abstract 48-1-53.)

5. Stefan Bergman: *On operators in the theory of partial differential equations and their application.* (Abstract 48-1-31.)

6. R. H. Bruck: *Isotopy of algebras with a principal unit.* (Abstract 48-1-8.)

7. M. F. Smiley and W. R. Transue: *Metric lattices as singular metric spaces* (Abstract 48-1-20.)

8. Marshall Hall and R. P. Dilworth: *The imbedding problem for modular lattices.* (Abstract 48-1-12.)

9. C. C. MacDuffee: *On the composition of algebraic forms.* (Abstract 48-1-15.)

10. Tomlinson Fort: *Generalizations of the Bernoulli polynomials and numbers and corresponding summation formulas.* (Abstract 47-11-456.)

11. I. A. Barnett and C. W. Mendel: *A property of the traces of a square matrix.* (Abstract 48-1-5.)

12. Saunders MacLane and O. F. G. Schilling: *Groups of algebras over an algebraic number field.* (Abstract 48-1–16.)

13. R. B. Kershner: *The continuity of functions of many variables.* (Abstract 48-1-39.)

14. Mark Kac: *On convergence of certain series of functions.* (Abstract 48-1-38.)

15. W. R. Transue: *Contributions to the theory of subharmonic functions.* (Abstract 48-1-59.)

16. J. W. Bradshaw: *On a certain class of continued fractions.* (Abstract 48-1-33.)

17. Otto Szász: *On a theorem of Hardy and Littlewood.* (Abstract 48-1-56.)

18. E. F. Beckenbach and Maxwell Reade: *Mean-values and harmonic polynomials.* (Abstract 48-1-30.)

19. J. L. Vanderslice: *Invariant theory of vector pencil fields.* (Abstract 48-1-79.)

20. J. W. Peters: *The euclidean geometry of the n-dimensional simplex.* (Abstract 48-1-78.)

21. André Weil and C. B. Allendoerfer: *A general proof of the Gauss-Bonnet theorem.* (Abstract 48-1-80.)

22. W. W. Flexner: *Noncommutative chains.* II. Preliminary report. (Abstract 48-1-90.)

23. G. W. Whitehead: *Homotopy groups of spheres and their rotation groups.* (Abstract 48-1-101.)

24. Harlan C. Miller: *On irreducible continua.* (Abstract 48-1-95.)

25. J. L. Kelley: *Symmetric product spaces.* Preliminary report. (Abstract 48-3-139.)

26. P. M. Whitman: *Note on a certain representation of lattice elements.* Preliminary report. (Abstract 48-1-24.)

27. Feodor Theilheimer: *The potential of curvilinear distributions.* (Abstract 48-1-72.)

28. J. L. Synge: *On the theory of the airfoil of finite span.* (Abstract 48-1-71.)

29. A. E. Heins: *On the transformation theory of the solution of partial differential equations.* II. Preliminary report. (Abstract 48-1-65.)

30. Willy Prager: *Fundamental theorems of a new mathematical theory of plasticity.* (Abstract 48-1-70.)

31. Alexander Weinstein: *On the flexural center and the center of twist.* (Abstract 48-1-74.)

32. R. E. Gaskell: *On longitudinal vibrations of a bar.* (Abstract 48-1-64.)

33. Henry Wallman: *On the reduction in harmonic distortion due to high frequency pre-emphasis.* Preliminary report. (Abstract 48-3-123.)

34. Alonzo Church: *On sense and denotation.* (Abstract 48-1-83.)

35. S. C. Kleene: *On the interpretation of intuitionistic number theory.* (Abstract 48-1-85.)

36. G. D. W. Berry: *On formalizing semantics.* (Abstract 48-1-82.)

37. Nelson Goodman: *Sequences.* (Abstract 48-1-84.)

38. Barkley Rosser: *The Burali-Forti paradox.* (Abstract 48-1-86.)

39. E. C. Berkeley: *Application of symbolic logic to punch card operations.* (Abstract 48-1-81-*t*.)

40. I. S. Cohen: *A generalization of Macauley's theorem on unmixed ideals.* (Abstract 48-1-9.)

41. Saunders MacLane and Samuel Eilenberg: *A theorem on group extensions.* (Abstract 48-1-94.)

42. G. Y. Rainich: *Factorization of polynomials, in a ring, with application to partial differential equations.* Preliminary report. (Abstract 48-1-51.)

43. Salomon Bochner and R. S. Phillips: *Absolutely convergent Fourier series in noncommutative normed rings.* (Abstract 48-1-6.)

44. R. M. Thrall and C. J. Nesbitt: *On the modular representations of the symmetric group.* (Abstract 48-1-21.)

45. Reinhold Baer: *Inverses and zero-divisors.* (Abstract 48-1-4-*t*.)

46. E. F. Beckenbach: *Painlevé's theorem and the analytic prolongation of a minimal surface.* (Abstract 48-1-29-*t*.)

47. R. H. Bruck: *Certain numerical invariants of polyadics.* (Abstract 48-1-7-*t*.)

48. Leonard Carlitz: *q-Bernoulli numbers and polynomials.* (Abstract 48-1-26-*t*.)

49. N. A. Court: *On the theory of the tetrahedron.* (Abstract 48-1-77-*t*.)

50. L. L. Dines: *On the mapping of n quadratic forms.* (Abstract 48-3-104-*t*.)

51. J. L. Doob: *Topics in the theory of Markoff chains.* (Abstract 48-1-34-*t*.)

52. H. J. Ettlinger: *The theory of the Riesz integral.* (Abstract 48-1-35-*t*.)

53. Abe Gelbart: *On functions of two complex variables with bounded real parts.* (Abstract 48-1-36-*t*.)

54. D. W. Hall: *On a theorem of E. E. Betz.* (Abstract 48-1-91-*t*.)

55. F. B. Hildebrand: *Note on the integro-differential equation of a problem in the theory of plane stress.* (Abstract 48-1-66-*t*.)

56. F. B. Jones: *A certain non-metric Moore space.* (Abstract 48-3-138-*t*.)

57. Irving Kaplansky and O. F. G. Schilling: *Some remarks on relatively complete fields.* (Abstract 48-1-14-*t*.)

58. H. N. Laden: *An interpolation polynomial involving derivatives of a prescribed function.* (Abstract 48-1-40-*t*.)

59. Joseph Lehner: *The Ramanujan identities and congruences for powers of eleven.* Preliminary report. (Abstract 48-1-27-*t*.)

60. E. J. McShane: *On Perron integration.* (Abstract 48-1-41-*t*.)

61. E. J. McShane: *The derivative of the indefinite Lebesgue integral.* (Abstract 48-1-42-*t*.)

62. S. B. Myers: *An existence theorem for a self-adjoint system of second-order, linear, homogeneous differential equations.* (Abstract 48-1-46-*t*.)

63. N. M. Oboukhoff: *The historical development of total differential as the principal part of the increment of a function of several variables.* (Abstract 48-1-47-*t*.)

64. J. F. Paydon and H. S. Wall: *An extension of the Stieltjes continued fraction theory.* (Abstract 48-1-49-*t*.)

65. Samuel Perlis: *Normal bases of cyclic fields of prime-power degree.* (Abstract 48-1-18-*t*.)

66. Maxwell Reade: *Some remarks on subharmonic functions.* Preliminary report. (Abstract 48-1-52-*t*.)

67. W. H. Roever: *Geometric statement of a fundamental theorem for four-dimensional orthographic axonometry.* (Abstract 48-3-133-*t*.)

68. O. F. G. Schilling: *Normal extensions of relatively complete fields.* (Abstract 48-1-19-*t*.)

69. H. M. Schwartz: *On sequences of Stieltjes integrals.* (Abstract 48-1-55-*t*.)

70. A. R. Schweitzer: *On the genesis of the algebra of logic in the foundations of geometry.* (Abstract 48-1-87-*t*.)

71. M. F. Smiley: *Elementary similarity transformations and the rational canonical form of a matrix.* (Abstract 48-3-107-*t*.)

72. R. H. Sorgenfrey: *Some theorems on co-terminal arcs.* (Abstract 48-1-98-*t*.)

73. E. W. Titt: *A method for integrating the linear hyperbolic equation in three independent variables.* (Abstract 48-1-58-*t*.)

74. W. J. Trjitzinsky: *Analytic theory of parametric linear partial differential equations.* (Abstract 48-1-60-*t*.)

75. S. M. Ulam: *A geometrical approach to the theory of representations of topological groups.* Preliminary report. (Abstract 48-1-61-*t*.)

76. A. D. Wallace: *Chains and structure of continua.* (Abstract 48-3-140-*t*.)

77. T. L. Wade and R. H. Bruck: *The characteristic function of a 2p-tensor.* (Abstract 48-1-22-*t*.)

78. P. A. White: *R-regular convergence spaces.* (Abstract 48-1-100-*t*.)

79. L. R. Wilcox: *Extensions of semi-modular lattices.* III. (Abstract 48-1-25-*t*.)

T. R. HOLLCROFT,
Associate Secretary

APPENDIX

Excerpts from Report of Treasurer[1]

December 15, 1941

To the Board of Trustees of the
American Mathematical Society

Gentlemen:

I have the honor to submit herewith the report of the Treasurer for the fiscal year ended November 30, 1941.

Surplus account shows a balance of $9,048.08 as compared with $6,966.39 at the beginning of the year. Two of the facts which contributed to this unanticipated increase may be mentioned here: (1) A considerable gain in the membership of the Society resulted in additional revenue from Initiation Fees and Dues; (2) Income from Investments exceeded the estimate by more than $1,000.

The market value on November 29, 1941 of the securities in the Pool was $7,613.09 less than book value. This is to be compared with the total amount, $4,201.96, held in accounts Reserve for Investment Losses and Profit and Loss on Sale of Securities. The market value on November 29, 1941 of securities in General and Restricted Funds was $963.00 less than book value. Profit on the sale of securities for General and Restricted Funds amounting to $295.73 is being carried as a reserve against losses to these funds.

Income earned by General and Restricted Fund Investments during the year amounted to $1,547.08. This represents a return of 1⅛% on amounts in savings banks, and approximately 4% on securities. Investment Income earned by the Pool amounted to $7,317.85, representing a return of approximately 4¼%. Total Investment Income from all sources was thus $8,864.93, corresponding to a yield of approximately 4¼%. This income and yield may be compared with $6,909.41 and approximately 4% for the fiscal year 1940.

The operating account for Mathematical Reviews continues to show receipts in excess of disbursements. However, this excess is due principally to advance subscriptions.

The Endowment Fund Principal, unchanged during the year, remains on the books at $60,110.68, representing a depreciation of $10,395.39 from a former value of $70,506.07.

During the year final settlement of the Estate of Marion Reilly brought the Society an additional sum of $6,738.78. A gift of $1,000 from Dr. Robert Henderson was also received. As directed by the Trustees, the gifts have been added to the Principal Funds in the Pool.

Respectfully submitted,
Bennington P. Gill, *Treasurer*

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[1] The report of the Treasurer for fiscal 1940 will be published in September 1942 with the biennial list of members.

BALANCE SHEET

Assets

	November 30, 1941	December 1, 1940
GENERAL AND RESTRICTED FUND ASSETS:		
Cash	$ 19,380.05	$ 14,624.30
Investments	28,814.24	21,393.87
	$ 48,194.29	$ 36,018.17
ENDOWMENT AND OTHER NONEXPENDABLE FUND ASSETS:		
Cash	$ 1,156.70	$ 659.28
Investments	160,395.09	152,549.20
	$161,551.79	$153,208.48
TOTAL ASSETS	$209,746.08	$189,226.65

Liabilities

GENERAL FUNDS:		
Sinking Fund	$ 420.09	$ 290.80
Profit on Sales of Securities	295.73	
Surplus	9,048.08	6,966.39
	$ 9,763.90	$ 7,257.19
RESTRICTED FUNDS:		
Bôcher Fund Income	$ 597.13	$ 540.96
Brown Fund Income	101.76	54.47
Cole Fund Income	659.39	560.41
Henderson Fund Income	21.63	
Moore Fund Income	1,583.06	1,483.73
Reilly Fund Income	2,650.05	2,280.84
Reilly Fund Principal	3,968.13	4,280.16
Bulletin Reprinting Fund	685.23	457.61
Transactions Reprinting Fund	103.63	103.63
Colloquium	5,187.14	4,718.95
Mathematical Reviews	16,924.45	8,426.11
Congress	5,943.44	5,853.67
War Preparedness	5.35	
	$ 38,430.39	$ 28,760.98
	$ 48,194.29	$ 36,018.17
ENDOWMENT AND OTHER NONEXPENDABLE FUNDS (POOL):		
Endowment Fund Principal	$ 60,110.68	$ 60,110.68
Bôcher Fund Principal	1,188.00	1,188.00
Brown Fund Principal	1,000.00	1,000.00
Cole Fund Principal	2,093.13	2,093.13
Henderson Fund Principal	1,000.00	
Moore Fund Principal	2,100.62	2,100.62
Reilly Fund Principal	19,553.32	12,502.07
Life Membership Reserve	5,191.30	5,507.90
Life Subscription Reserve	112.78	114.09
Colloquium	5,000.00	5,000.00
Mathematical Reviews	60,000.00	60,000.00
Reserve for Investment Losses	2,728.24	1,926.26
Profit on Sales of Securities	1,473.72	1,665.73
	$161,551.79	153,208.48
TOTAL LIABILITIES	$209,746.08	$189,226.65

SUMMARY STATEMENT OF INCOME AND EXPENDITURES
1940–1941

	1941		1940	
	Receipts	*Disbursements*	*Receipts*	*Disbursements*
GENERAL RECEIPTS				
Dues	$14,970.19		$14,380.96	
Dues—Contributing Memberships	1,040.00		1,114.92	
Dues—Institutional Memberships	6,468.31		6,705.00	
Initiation Fees	515.25		584.75	
Investment Income	4,147.27		3,191.46	
Miscellaneous	343.15		695.95	
GENERAL DISBURSEMENTS				
Secretaries		$ 6,091.29		$ 5,800.00
Treasurer		750.00		650.00
Officers Traveling		888.93		749.54
Library		824.50		629.82
Committee Expenses		101.04		211.24
Office Furniture and Fixtures		51.70		31.00
Gibbs Lecture				19.88
Emergency		277.29		287.86
Total	$27,484.17	$ 8,984.75	$26,673.04	$ 8,379.34
Excess of General Receipts	$18,499.42		$18,293.70	
PUBLICATION				
Bulletin	$ 1,755.47	$11,037.14	$ 1,631.65	$12,405.21
Bulletin Reprinting	227.62		145.85	
Transactions	4,816.11	8,968.10	5,411.24	8,521.05
Transactions Reprinting				116.80
Colloquium	3,437.74	2,969.55	3,121.11	2,727.54
Mathematical Reviews	21,467.56[1]	12,969.22	20,302.06[2]	14,356.77
Semicentennial Publications	129.29		141.30	
American Journal		2,500.00		2,500.00
Total	$31,833.79	$38,444.01	$30,753.22	$40,627.37
Excess of Cost of Publication		$ 6,610.22		$ 9,874.15
MISCELLANEOUS				
Profit on Sales of Securities	$ 103.72		$ 1,578.49	
Adjustment for Depreciation of Value of Investments			87.24	$24,804.97
Special Funds	9,526.51[3]	$ 1,000.00	1,366.89	487.10
Total	$ 9,630.23	$ 1,000.00	$ 3,032.62	$25,292.07
Difference	$ 8,630.23			$22,259.45
Net change in assets	$20,519.43			$13,839.90
ASSETS BEGINNING OF YEAR	$189,226.65		$203,066.55	
ASSETS END OF YEAR	$209,746.08		$189,226.65	

[1] Includes $6,809.57 receipts for subscriptions for 1942–1944 and $1,000 appropriation from Reilly Fund Income.
[2] Exclusive of $1,000 appropriation from Surplus.
[3] Includes $7,738.78 principal of gifts.

JAMES WATERMAN GLOVER—IN MEMORIAM

James Waterman Glover, emeritus professor of mathematics at the University of Michigan, died in Ann Arbor, Michigan, on July 15, 1941. In his passing, actuarial and statistical science has lost one of its most distinguished pioneers.

Professor Glover was born at Clio, Michigan, on July 24, 1868. He was graduated from the University of Michigan with the class of 1892, where he came under the influence of F. N. Cole, and received his doctor's degree from Harvard University in 1895, being the first one to do his thesis work under the supervision of Maxime Bôcher; his thesis subject was *Properties of Solutions of the Partial Differential Equation* $\Delta y+ky=0$. He then returned to the University of Michigan as a member of the Department of Mathematics, holding the various ranks, Assistant Professor in 1903, Junior Professor in 1906, and Professor in 1911, until he retired in 1938. He was chairman of the Department of Mathematics from 1927 to 1934. During the years 1930 to 1932 he had leave of absence, and served as president of the Teachers' Insurance and Annuity Association of America. He was a member of the Council of the American Mathematical Society from 1927–1929.

Beginning in 1902 with a course in actuarial theory, Professor Glover gradually developed courses in the mathematics of finance, life insurance, and statistics, at the University of Michigan. As a consequence of the training afforded by these courses, he was able to place hundreds of students in responsible positions with life insurance companies and state departments. At the time of his retirement about two hundred of his former students, many of them executives of major insurance companies, gave a dinner in Chicago honoring him. At this meeting a fellowship in insurance, bearing his name, was established.

In his administrative capacity he not only built up the work in actuarial mathematics and mathematical statistics at the University, but was also responsible for greatly strengthening the work of the Department in other branches by the addition of promising young men.

His activities outside of the University were numerous. He served as consulting expert for the Canadian Royal Commission on Insurance in 1896, for the Wisconsin Legislative Investigation Committee in 1906, for the Wisconsin Joint Committee on Banks and Insurance in 1907, and for numerous insurance companies. From 1910 to 1929 he served the U. S. Census Bureau as Expert Special Agent and dur-

ing this time the United States Life Tables were prepared under his supervision.

While Professor Glover was a forceful person so far as being able to accomplish something he thought ought to be done, he was an extremely pleasant person with whom to be associated. One could oppose something he proposed to do, feeling certain it would not affect a personal friendship. He was fond of hunting and fishing and was a delightful companion on an outing. His accomplishments are a lasting monument to his memory.

Following is a list of his publications of mathematical interest:

1. *On groups whose orders are products of three prime factors* (with F. N. Cole), American Journal of Mathematics, vol. 15 (1893), pp. 191–220.

2. *Derivation of the United States mortality table by osculatory interpolation*, Quarterly Publication of the American Statistical Association, (n.s.), vol. 12 (1910), pp. 85–109.

3. *A general formula for the valuation of securities*, American Mathematical Monthly, vol. 22 (1915), pp. 82–88.

4. *Some formulas connected with the calculation of annual dividends on the contribution plan*, Record of the American Institute of Actuaries, vol. 5 (1916), pp. 37–43.

5. *United States Life Tables*: 1910, Government Printing Office, Washington, 1916, 65 pp., 25 life tables.

6. *United States Life Tables*, 1890, 1901, 1910 *and* 1901–1910, Government Printing Office, Washington, 1921, 496 pp., 22 diagrams, 52 graphs.

7. *Tables of Compound Interest Functions and Logarithms of Compound Interest Functions* (with H. C. Carver), Ann Arbor, 1921, 77 pp.

8. *Tables of Applied Mathematics in Finance, Insurance, Statistics*, Ann Arbor, 1923, 13+676 pp.

9. *Tables of Statistical Data for the Use of Students of Mathematical and Economic Statistics* (with H. C. Carver), Ann Arbor, 1923, 53 pp.

10. *Interpolation, summation and graduation*, chap. 3, pp. 34–61, *Handbook of Mathematical Statistics*, Houghton Mifflin, 1924.

11. *Quadrature formulas when ordinates are not equidistant*, Proceedings of the International Congress of Mathematicians, Toronto, 1924, vol. 2.

12. *Report on statistical teaching in American colleges and universities*, Journal of the American Statistical Association, vol. 21 (1926).

13. *An Introduction to the Mathematics of Life Insurance* (with W. O. Menge), New York, Macmillan, 1935, 9+190 pp.

PETER FIELD

BOOK REVIEWS

Tables of Probability Functions. Vol. I. New York, Work Projects Administration. 1941. 28+302 pp. $2.00.

The earliest systematic table of the error function or probability integral was prepared by James Burgess and published in the Transactions of the Royal Society of Edinburgh, volume 39, Part II, 1898, pp. 257–321. This was followed by that compiled by W. F. Sheppard under the auspices of the British Association for the Advancement of Science, published by the Cambridge University Press in 1939, and by various other minor tables.

The present Tables extend the range of all existing ones and provide a smaller tabular interval. The usual control methods employed by the Agency to make them accurate have been used.

Both functions have been calculated to fifteen decimal places at intervals of 0.0001 in the range between 0 and 1 and at intervals of 0.001 in the range from 1 to 5.6. A short supplementary table (Table II) is included giving the values of $H(x)$ and its derivative to eight significant figures for x ranging from 4 to 10 at intervals of 0.01. The procedures for direct and inverse interpolation are explained and the degree of approximation attained are emphasized. The method employed for the actual computation is outlined together with the checks for control of accuracy.

The pages are 9 by 4 inches, both $H'(x)$ and $H(x)$ appearing in contiguous columns. There are 51 lines on a page in blocks of five, the last entry on a page being repeated at the top of the following one.

VIRGIL SNYDER

Guide to Tables in the Theory of Numbers. By Derrick Henry Lehmer. (Bulletin of the National Research Council, Number 105. Division of Physical Sciences, Committee on Mathematical Tables and Aids to Computation, Report 1: Report of the Subcommittee on Section F: Theory of Numbers.) Washington, D. C., National Research Council, 1941. 14+177 pp. $2.50.

In a foreword Professor R. C. Archibald, Chairman of the Committee, states that "In broad outline it exhibits the general plan for all Reports in the series. In adopting this plan the Committee desires to make clear that the Reports are being prepared primarily for scholars and others active in scientific work throughout the world."

Directions for the use of the Report are given in the Introduction.

This also contains an explanation of the practical point of view adopted in deciding what constituted a table in the theory of numbers and what tables were worthy of inclusion. The omissions include old obscure tables which have been superseded by more extensive and more easily available ones, and short tables in which every entry may be easily computed.

The rest of the Report is in three parts:

I. Descriptive Survey.
II. Bibliography.
III. Errata.

In Part I, which is about 80 pages in length, topics in the theory of numbers are classified under 17 headings **a**–**q**, with subheadings indicated by subscripts. Under each heading there is a description of what tables are listed in this topic and what they contain. The pages are numbered at the bottom and the topic described on a page is indicated by the appropriate letter of classification at the top.

With each table mentioned in Part I there appears the author's name followed by a number which refers to the complete bibliographic reference of Part II. Here the material is arranged alphabetically by authors. Following each reference a letter (with or without a subscript) in square brackets indicates the nature of the tables contained in the work referred to. In addition, libraries in which the work may be found are given in the coding used by the *Union List of Serials.* For this purpose 37 representative libraries were selected, the list and key to the code being given following the foreword of the book.

Thus it will be seen that if either the subject matter or the author's name is known the location of the table is a simple matter. This feature in itself makes the book of great value, but even this is overshadowed by Part III which collects for the first time the list of errors which have been discovered in the tables. The authority for corrections and a reference to the source, if published, are usually given after the errors. Tables in which errors have been found are indicated in Part II by an asterisk.

This report is indeed a guide to tables in the theory of numbers and it is, moreover, one which can be followed with ease.

R. D. JAMES

Waves. A Mathematical Account of the Common Types of Wave Motion. By C. A. Coulson. Edinburgh, Oliver and Boyd, 1941. 156 pp. $1.50.

This book will be welcomed by those who are interested in an ele-

mentary introduction to the subject of wave motion; it will also interest the experts in some one type of physical wave motion as a book for ready reference concerning other types of physical wave motion. In common with other books belonging to "University Mathematical Texts" this book is remarkably informative for its size. In the first chapter the wave equation and its principal solutions are introduced. From there on are treated in succession waves on strings, waves in membranes, longitudinal waves in bars and springs, waves in liquids, sound waves, and electric waves. The last chapter contains some general considerations. In each chapter the equations for the particular type of waves and the boundary conditions are derived; the methods of solution are illustrated by well-chosen examples. The presentation is clear and straightforward. Each chapter is followed by problems.

In the words of the author, "The object of this book is to consider from an elementary standpoint as many different types of wave motion as possible. In almost every case the fundamental problem is the same, since it consists in solving the standard equation of wave motion; the various applications differ chiefly in the conditions imposed on these solutions. For this reason it is desirable that the subject of waves should be treated as one whole, rather than in several distinct parts; the present tendency is in this direction." If one is to criticize the book in this connection, it is, perhaps, in order to suggest that the announced purpose could be served still better by giving the impedance concept the place it rightly deserves in wave theory. The original wave equation usually consists of two first order equations connecting the force and the velocity (or displacement). The solution will consist of a wave of force and a wave of velocity. By placing emphasis on this "two-wave" aspect, greater uniformity in treatment of reflection can be attained. It is hoped that the author will consider this point of view in the next edition.

SERGEI A. SCHELKUNOFF

Lectures in Topology. The University of Michigan Conference of 1940. Edited by R. L. Wilder and W. L. Ayres. Ann Arbor, University of Michigan Press, 1941. 316 pp. $3.00.

This volume is a collection of the papers presented at the University of Michigan Conference on Topology in June 1940. The scope of the book is indicated by the following list of titles of the longer papers:

Solomon Lefschetz, Abstract complexes; R. L. Wilder, Uniform lo-

cal connectedness; N. E. Steenrod, Regular cycles of compact metric spaces; Samuel Eilenberg, Extension and classification of continuous mapping; Hassler Whitney, On the topology of differentiable manifolds; S. S. Cairns, Triangulated manifolds and differentiable manifolds; P. A. Smith, Periodic and nearly periodic transformations; Leo Zippin, Transformation groups; Saunders MacLane and V. W. Adkisson, Extensions of homeomorphisms on the sphere; O. G. Harrold, Jr., The role of local separating points in certain problems of continuum structure; L. W. Cohen, Uniformity in topological space; E. W. Chittenden, On the reduction of topological functions. There are also short accounts of nine other papers.

As can be seen from this list, practically every phase of modern topology is touched upon in this collection. Many of the papers are of a discursive nature, with most of the proofs omitted, and so the total amount of ground covered is quite extensive. We heartily recommend this book to any worker in topology as an excellent source of information on the present status of this subject.

R. J. WALKER

An Introduction to Linear Transformations in Hilbert Space. By F. J. Murray. (Annals of Mathematics Studies, no. 4.) Princeton University Press, 1941. 135 pp. $1.75.

The purpose of this book, according to the author, is "to present the most elementary course possible on this subject" and at the same time "to emphasize those notions which seem to be proper to linear spaces." Despite the assertion that these aims are not antagonistic, the exposition would be pretty tough going for the average graduate student. Although the reader is not assumed, except in an isolated section, to know about Lebesgue integration, and although the proof of such a comparatively elementary fact as that a continuous image of a compact set is compact is given in detail (p. 48), many parts of the book assume a great deal more sophistication.

The discussion is almost entirely unmotivated: the beginner might like to know *why* one studies spectral families, or the adjoints of operators. Even to one familiar with the theory it requires proof that von Neumann's definition of T^* is equivalent to the easier one usually given for bounded transformations; T^* is defined as the negative of the transformation whose graph is the orthogonal complement of the graph of T.

Concerning the author's choice of the order of the material, it is questionable whether or not it is pedagogically advisable to aim the

discussion at unbounded operators from the very beginning, particularly since the main theorem (the spectral representation of self adjoint operators) is first proved for the bounded case. It would seem better to the reviewer to expound all the easy theory of bounded operators first, and thus prepared to call attention to the delicate considerations necessary to study the unbounded case. Also, in Chapter II we find practically the same proof used twice, once to establish the Riesz theorem on the representation of a bounded linear functional by an inner product (Theorem IV), and once to prove the possibility of projection on any closed linear manifold in Hilbert space (Theorem VI). The extremely elementary derivation, due to Riesz (Acta Szeged, vol. 7 (1934–1935), p. 37), of the former from the latter, could have been used here to good advantage.

The book contains many minor slips and typographical errors which may cause serious confusion. Thus in the statement of the axioms for a linear space (p. 4) the assumption $1f=f$ is omitted, and on p. 34, Theorem I, which is stated for an arbitrary transformation, is proved by reference to a lemma valid only for the additive case. Regrettable also is the author's reluctance to give to well known theorems their usual names: Schwarz's inequality, Bessel's inequality, Parseval's identity, and the Riesz-Fischer theorem are all indiscriminately referred to as Theorem n of Chapter m.

In Chapter VII (footnotes, pp. 67–68) there are some pretty examples of spectral families, and the Hellinger integral is treated courageously from the modern point of view and not reduced by means of weak convergence to the classical numerical case. The last two chapters are an excellent idea, carried out unfortunately too rarely: they contain quick sketches of further developments and applications, and references to the literature. On the whole the book is a compact and unified presentation of a well defined part of Hilbert space theory, and as such may appeal to the reader interested in learning only that part of the theory which even a non-specialist often needs.

PAUL R. HALMOS

NOTES

Brown University is continuing during the summer (June 15–August 29, 1942) and the next academic year the Program of Advanced Instruction and Research in Mechanics already in operation for a year. In the summer, 80 participants will be accepted (of which 20 will be engaged entirely in research) and in the academic year 40. Since this program is supported by the United States Office of Education, the Carnegie Corporation of New York, and the Rockefeller Foundation, no fees will be charged. The faculty for the summer consists of Stefan Bergman, Léon Brillouin, Willy Feller, Richard von Mises, Willy Prager, S. A. Schelkunoff, I. S. Sokolnikoff, and J. D. Tamarkin, who are offering a dozen courses with opportunities for research in various branches of applied mathematics, chiefly in connection with mechanics. For the next academic year eight courses are scheduled. Several substantial fellowships are available for highly qualified participants. Information in regard to each of these programs may be obtained from the Dean of the Graduate School, Brown University, Providence, R. I.

A feature of the program for the coming summer quarter at the University of Chicago will be a seminar on the calculus of variations. Professor Emeritus G. A. Bliss will return to the University to conduct this seminar. Other members of the summer staff who will participate in the seminar include Professors Tibor Radó of Ohio State University, M. F. Smiley of Lehigh University, and L. M. Graves, M. R. Hestenes, and W. T. Reid. Those who are interested may secure further information by writing to Professor Graves. Professor Radó will also give a course on subharmonic functions, and as part of the defense program, Professor Reid will give a course on exterior ballistics.

Connecticut College is establishing a scholarship fund in honor of the late Professor D. D. Leib.

It is planned to establish an Institute of Applied Mathematics at New York University.

Professor E. A. Milne of the University of Oxford has been awarded a Royal Medal by the Royal Society for his researches on the atmospheres of the earth and the sun, on the internal constitution of the stars, and on the theory of relativity.

During 1941 several Russian mathematicians were honored by

awards of Stalin Prizes. These prizes were established in honor of the sixtieth birthday of Joseph Stalin, and are of three classes: Frist Prize 100,000 rubles, Second Prize, 50,000 rubles, Third Prize, 25,000 rubles. First Prizes were awarded to Mr. I. M. Vinogradov for his work *A new method in the analytical theory of numbers*; to Mr. P. A. Gelvich for his works *On dispersion, probability of hits, and mathematical anticipation of the number of hits, Theoretical fundamentals for the elaboration of gunnery rules*, and *Firing at rapidly moving targets*; to Mr. P. L. Kapitza for his work *Turbodetander for obtaining low temperatures and its application for obtaining liquid air*; to Mr. N. I. Muskhelishvili for his work *Some basic problems in the mathematical theory of elasticity*. Second Prizes were awarded to Mr. A. I. Alikhanov and Mr. A. I. Alikhanian for their scientific work on radio; to Mr. A. N. Kholmogorov and Mr. A. Y. Khinchin for their works *Asymptotic laws of the theory of probability, On analytical methods in the theory of probability*, and *Limit laws of the sums of independent chance quantities*; to Mr. L. S. Pontriagin for his work *Topological groups*; and to Mr. S. L. Sobolev for his works *Some questions in the theory of the expansion of vibrations* and *On the theory of non-linear hyperbolic equations with partial derivatives*.

Professor Theodore von Kármán of California Institute of Technology has been awarded the American Society of Chemical Engineers Medal for 1941.

Dr. W. H. Garrett, for thirty-nine years head of the department of mathematics and astronomy at Baker University, has been awarded the doctor of science degree by Illinois College.

Professor C. J. Blackall of the College of St. Thomas has been appointed to a professorship at De Sales College, Toledo, Ohio.

Dr. B. W. Brewer of the Agricultural and Mechanical College of Texas has been promoted to an assistant professorship.

Associate Professor C. M. Cleveland of the University of Texas has been promoted to a professorship in applied mathematics.

Dr. T. J. Higgins of Purdue University has been appointed to an assistant professorship at Tulane University.

Dr. E. E. Ingalls of Albion College has been promoted to an assistant professorship.

Dr. F. B. Jones of the University of Texas has been promoted to an assistant professorship.

Dean Mary N. Keith of the University of Redlands has retired.

Professor J. R. Kline of the University of Pennsylvania has been appointed Thomas A. Scott Professor of Mathematics.

Assistant Professor E. C. Klipple of the Agricultural and Mechanical College of Texas has been promoted to an associate professorship.

Mr. Neil Little of Purdue University has been promoted to an assistant professorship.

Assistant Professor G. A. Lyle of the U. S. Naval Academy has been promoted to an associate professorship.

Reverend P. H. McGrath of St. Peter's College, Jersey City, New Jersey, has been promoted to a professorship.

Dr. C. A. Messick of Oakland City College, Oakland City, Indiana, has been appointed to a professorship and will be head of the department of mathematics.

Assistant Professor J. E. Powell of Michigan State College has been promoted to an associate professorship.

Dr. S. M. Ulam of the University of Wisconsin has been promoted to an assistant professorship.

Miss Margaret C. Weeber of the Teachers College of Connecticut has been promoted to an assistant professorship.

Dr. H. N. Wright of the College of the City of New York will succeed Dr. F. B. Robinson as president of the College.

Dr. R. B. Kershner of Johns Hopkins University has been made an associate in mathematics.

Dr. H. H. Campaigne of the University of Minnesota is on leave to serve in the U. S. Navy.

Professor W. A. Hurwitz of Cornell University is on leave of absence at the Institute for Advanced Study.

Professor Eugenie M. Morenus of Sweet Briar College is on leave for this academic year.

Professor J. F. Ritt of Columbia University is on leave of absence for the spring term.

Dr. E. N. Shawhan of the University of Minnesota is on leave to do defense work in the Naval Ordnance Laboratory at Washington.

The following appointments to instructorships are announced: Central College, Fayette, Missouri: Dr. P. B. Burcham; Haverford College: Dr. L. C. Green, Dr. André Weil; Illinois Institute of Technology: Dr. Herman Meyer; State University of Iowa: Dr. W. D. Berg; University of Maine: Dr. Esther Comegys; University of Minnesota: Dr. Abraham Spitzbart; Montana State College: Dr. C. B. Smith; Agricultural and Mechanical College of Texas: Mr. C. H. Cunkle, Mr. W. B. Evans, Mr. H. W. Grant, Mr. J. R. Smith.

Mr. I. O. Griffith of the University of Oxford died September 22, 1941.

The death of Professor Tullio Levi-Civita of the University of Rome has been announced. He had been a member of the Society since 1904.

Professor Emile Picard, Secretary of the French Academy of Sciences, died December 12, 1941, in Paris. He was eighty-five years old.

Professor C. S. Atchison of Washington and Jefferson College died November 21, 1941, at the age of fifty-nine years. He had been a member of the Society since 1907.

It is reported that Dr. J. D. Barter lost his life at sea through enemy action.

The death of Mr. Robert Coleman of Wilberforce University has been reported.

Dean Emeritus W. P. Durfee of Hobart College died December 17, 1941, at the age of eighty-six years. He had been a member of the Society since 1891.

Professor Emeritus G. I. Gavett of the University of Washington died December 22, 1941.

Professor E. J. Hirschler of Bluffton College died May 22, 1941, at the age of sixty-five years. He had been a member of the Society since 1926.

Professor U. G. Mitchell of the University of Kansas died January 1, 1942, at the age of sixty-nine years. He had been a member of the Society since 1909.

Professor C. M. Sparrow of the University of Virginia died August 30, 1941, at the age of sixty-one years. He had been a member of the Society since 1909.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

102. A. A. Albert: *Non-associative algebras. II. New simple algebras.*

It is first shown that non-associative algebras A with a unity quantity e and order n have the same properties for decomposition in direct sums as do associative algebras. Let G be any multiplicative group of order n of non-singular linear transformations S on A such that $eS=e$, H be a subset containing I, g be a set of $g_{S,T}$ in A defined for every S and T of G and which are not zero divisors. Then construct the crossed extension $E=(A, G, H, g)$. It is an algebra of order nm with e as unity quantity. For separable algebras A conditions are given that E be simple and central simple. It is always simple (central simple) when A is and when $H=[I]$. Then an iterative process results for extending ordinary crossed products of order r^2 to central simple algebras of order r^t which are necessarily non-associative. Every central simple algebra of order n may be extended by the use of an arbitrary permutation group on n letters and a class of permutation algebras is obtained. Finally, a list of fundamental unsolved problems is given. (Received January 7, 1942.)

103. R. A. Beaumont: *Projections of the prime-power abelian group of order p^m and type $(m-1, 1)$.*

A group H is the projection of a group G if there is a $(1-1)$ correspondence between the set of subgroups of G and the set of subgroups of H which preserves the partial ordering of the subgroups. Since R. Baer has given necessary and sufficient conditions that a group H be a projection of a group G which is the direct product of cyclic groups of order p, in the study of the projections of the prime-power abelian group G of order p^m and type $(m-1, 1)$, we may take $m>2$. It is shown that if $p>2$, the only group H, essentially different from G, which is a projection of G is the non-abelian group of order p^m containing an element of order p^{m-1}. If $p=2$ and $m>3$, the only group H, essentially different from G, which is a projection of G is the non-abelian group $\{U_1, U_2\}$ where U_1 and U_2 are subject to the sole defining relations: $U_1^{2^{m-1}}=U_2^2=1$, $U_2U_1U_2=U^{1+2^{m-1}}$. If $p=2$ and $m=3$, a group H is a projection of G if and only if H is isomorphic to G. (Received January 16, 1942.)

104. L. L. Dines: *On the mapping of n quadratic forms.*

A transformation $x_i=Q_i(z)$, $(i=1, 2, \cdots, n)$, in which each $Q_i(z)$ is a real quadratic form in the real variables $z^1, z^2, \cdots, z^m$, maps the m-dimensional z-space into a set of points $\mathfrak{M}$ in the n-dimensional x-space. The present paper considers properties

of this map $\mathfrak{M}$, and on the basis of these properties determines necessary and sufficient conditions for the existence of linear combinations $\sum \lambda_i Q_i(z)$ which are positive definite, and also conditions for the existence of such combinations which are semi-definite. The conditions are, for general n, naturally not so simple as those obtained for the case $n=2$ (this Bulletin, vol. 47 (1941), pp. 494–498). But they appear to be simpler as well as more comprehensive than those obtained by Finsler (Commentarii Mathematici Helvetici, vol. 9 (1937), pp. 188–192), and by Hestenes and McShane (Transactions of this Society, vol. 47 (1940), pp. 501–512) with which they may be compared. The paper will appear in this Bulletin. (Received December 17, 1941.)

105. H. B. Mann: *Proof of the conjecture on the density of sums of sets of positive integers.*

Let $A(n)$ denote the number of positive integers less than or equal to n in the set A. If $A+B=C$ then it is proved in this paper that: $C(n)/n \geqq \min\ (A(x)/x+B(x)/x)$ for $x \leqq n$. The conjecture $\gamma \geqq \alpha_1+\alpha_2+\cdots+\alpha_n$ is an immediate consequence of this result. Let $n_1, n_2, \cdots, n_r, \cdots$ be the gaps in C. Let B_r^+ be the numbers of B that are not of the form n_r-a with a in A or equal to 0. The theorem is equivalent to the following statement: If for any r, $rn_i > in_r$ and $rn_i > (B_i^+(n_i)+1)n_r$ for $i=1, 2, \cdots, r-1$, then $B_r^+(n_r) \geqq r-1$. Let $n_r-n_i=d_i$ then there always exist values i, j for which $n_r-d_i = a+b$ (a in A or equal to 0, b in B or equal to 0). Among all the b's found in this way the smallest is chosen and denoted by e_1. Let B^1 contain all numbers e_1+d_s and N^1 all numbers n_t that satisfy an equation $a+e_1+d_s=n_t$. The numbers of B^1 are added to the numbers in B and the process repeated. Thus sets $B^1, B^2, \cdots, B^i$ are formed and sets $N^1, N^2, \cdots, N^i$ and it is shown that this construction can be continued until all values n_t have been absorbed into the sets N^i. The sets B^i then contain $r-1$ different numbers of B_r^+. (Received January 15, 1942.)

106. A. R. Schweitzer: *Note on functions which generate an abstract field.*

In a previous report (this Bulletin, vol. 27 (1921), p. 249) the author stated that the following functions each generate, under suitable postulational assumptions, an abstract field: $(x+y)y$, $(1+x)y$, $(x-y)y$, $(1-x)y$. The former two functions were obtained as special instances of the function $(x+y)z$ which, it was stated, also generates a field. In this note a simple a priori proof is given by exhibiting chains of definitions of functions leading to the functions $x+y$, xy, in each instance. These chains are essentially as follows: I. $(x+y)y$: 0, 1, $x+1$, $-x$, $-x+1$, $x-1$, $-(x+y)$, $x+y$, xy. II. $(1+x)y$: 0, -1, $-(1+x)$, $-xy$, $-x$, xy, x/y, $x+y$. III. $(x-y)y$: 0, -1, $-(1+x)$, $-x$, $x-1$, $-x+1$, $x+1$, $x+y$, xy. IV. $(1-x)y$: 0, 1, $1-x$, xy, x/y, $x-y$, $x+y$. Reference is made to a report by the author, this Bulletin, vol. 26 (1920), p. 441. (Received December 29, 1941.)

107. M. F. Smiley: *Elementary similarity transformations and the rational canonical form of a matrix.*

If A is a square matrix with elements in a field F and E is an F-elementary transformation matrix, then $B=EAE^{-1}$ is said to be obtained from A by an elementary similarity transformation. A process is described involving a finite number of such transformations which replaces A by the matrix $\operatorname{diag}\{B^{(1)}, \cdots, B^{(t)}\}$ where the $B^{(i)}$ $(i=1, \cdots, t)$ are matrices in rational canonical form and the characteristic matrix of each $B^{(i)}$ has just one non-trivial invariant factor. The reduction of A to

rational canonical form is then easily obtained. Examples are given to support the contention that this process simplifies the computation of the rational canonical form of A. (Received December 15, 1941.)

ANALYSIS

108. G. E. Forsythe and A. C. Schaeffer: *A remark on Toeplitz matrices.*

A doubly infinite matrix (a_{mn}) is said to be regular if for every sequence $\{x_n\}$ with limit x' the corresponding sums $y_m=\sum_n a_{mn}x_n$ are defined for all m and have the limit x'. An apparently more general definition of regularity is that the sums defining y_m exist for all sufficiently large m, depending on $\{x_n\}$, and have the limit x'. Tamarkin (this Bulletin, vol. 41 (1935), pp. 241–243) has obtained necessary and sufficient conditions for the second type of regularity. This result is obtained by elementary methods and related topics are discussed. (Received January 23, 1942.)

109. H. L. Garabedian: *Hausdorff integral transformations.*

This paper involves a study of the integral transformation $v(x)=\int_0^x u(y)d\phi(y/x)$, defining a method of summation $(H, \phi(x))$, where $u(x)$ is bounded and continuous, $x\geqq 0$, and where $\phi(x)$ is either a Hausdorff mass function or satisfies the conditions: (i) $\phi(x)$ is of bounded variation on the interval $0\leqq x\leqq 1$, (ii) $\phi(x)$ is continuous on the interval $(0, 1)$ except possibly at $x=1$, (iii) $\phi(0)=0$, (iv) $\phi(1)=1$. It is proved that the transformation is regular when and only when $\phi(x)$ is a Hausdorff mass function, and sufficient conditions involving the Silverman-Schmidt integral equations are obtained in order that $(H, \phi_1(x))\supset(H, \phi_2(x))$, in the case that $\phi_1(x)$ and $\phi_2(x)$ satisfy the conditions stated above. These results are extensions of those obtained by Silverman (Transactions of this Society, vol. 26 (1924), pp. 101–112). (Received January 10, 1942.)

110. A. M. Gelbart: *Functions of two variables with bounded real parts in domains not equivalent to the bicylinder.*

Let $f(z_1, z_2)$ be regular in the interior of a finite four-dimensional domain $\mathfrak{M}^4$, bounded by certain analytic hypersurfaces, and in general not equivalent to the bicylinder, and let $f(z_1, z_2)$ have a bounded real part in $\mathfrak{M}^4$. These domains were first considered by Bergman, and are termed by him, domains with distinguished boundary surfaces. An upper bound for $|f(z_1, z_2)|$ is obtained in terms of only $\max Re\, f(z_1, z_2)$ in $\mathfrak{M}^4$, $f(0, 0)$ and the domain considered. From a formula for $\partial^{m+n}f(z_1, z_2)/\partial z_1^m\partial z_2^n$ in $\mathfrak{M}^4$, previously obtained by the author (Transactions of this Society, vol. 49 (1941), pp. 199–210), an upper bound is also obtained for $\left|\partial^{m+n}f(z_1, z_2)/\partial z_1^m\partial z_2^n\right|$, again in terms of only $\max Re\, f(z_1, z_2)$ in $\mathfrak{M}^4$, $f(0, 0)$ and the domain. These results depend upon the establishment of a form of the Schwarz lemma in $\mathfrak{M}^4$ for two variables. (Received January 29, 1942.)

111. H. J. Greenberg and H. S. Wall: *Hausdorff means included between* $(C, 0)$ *and* $(C, 1)$.

It is shown that if $\phi(u)$ is any function of bounded variation on the interval $0\leqq u\leqq+\infty$ such that $\phi(+\infty)-\phi(0)=1$, then the function $\alpha(z)=\int_0^\infty d\phi(u)/(1+zu)$ is a regular moment function; and that when $\phi(u)$ is further restricted to be monotone

then $(C, 0) \subset [H, \alpha(n)] \subset (C, 1)$. Conditions under which $[H, \alpha(n)]$ is equivalent to $(C, 0)$ or to $(C, 1)$ are obtained which are analogous to the conditions found by Scott and Wall (abstract 47-3-144) for the special case where $\phi(u) \equiv 1$ for $u \geqq 1$, $\phi(0) = 0$, namely $(C, 0) \approx [H, \alpha(n)]$ if and only if $\phi(+0) - \phi(0) > 0$, and $[H, \alpha(n)] \approx (C, 1)$ if and only if $\int_0^\infty d\phi(u)/u < \infty$. Certain transformations of moment sequences, for example, the Hausdorff transformation, are discussed. (Received January 14, 1942).

112. Walter Leighton and W. J. Thron: *On the convergence of continued fractions.*

In the $z = x + iy$ plane let new coordinate axes x', y' be obtained by rotating the original axes through an angle β. It is shown that if the elements a_n of the continued fraction $1 + K(a_n/1)$ are complex numbers and lie in a closed bounded region in the interior of one of the parabolic regions $y'^2 \leqq \cos^2\beta/2(x' + (1/4)\cos^2\beta/2), -\pi < \beta < \pi$, the continued fraction converges. Further it is established that the value of this continued fraction lies in the half-plane defined by the relation $(x - 1/2)\cos\beta/2 + y \sin\beta/2 \leqq 0$. (Received January 8, 1942.)

113. Walter Leighton and W. J. Thron: *On value regions of continued fractions.*

If the elements $a_n = \rho e^{i\varphi}$ of a continued fraction $1 + K(a_n/1)$ lie in a parabolic region $\rho \leqq 2d(1 - d)/(1 - \cos\theta)$ $(1/2 < d < 1)$ and if the a_n are bounded in absolute value, the continued fraction is known to converge. It is shown, that the value $z = Re^{i\theta}$ of this continued fraction lies in the region $R \geqq 2d(1 - d)/(1 - 2d + \cos\theta)$, $-\beta < \theta < \beta$; $\beta = \arccos(1 - 2d)$. Every value in this region is taken on by at least one continued fraction $1 + K(a_n/1)$ with elements in the described parabola. (Received December 12, 1941.)

114. A. N. Lowan, Gertrude Blanch, and William Horenstein: *Inversion of the q-series associated with Jacobi elliptic functions.*

In the computation of the Jacobi elliptic functions, sn, cn, and dn with the aid of the theta functions, it was found necessary to invert the expression $\epsilon = (1/2)(1 - k'^{1/2})/(1 + k'^{1/2}) = (1/2)\theta_2(0, q^4)/\theta_3(0, q^4)$, and thus obtain q as a power series of ϵ. Weierstrass had given the first four terms of this expansion (*Werke*, II (1895), p. 276); Milne-Thomson found two additional terms (Journal of the London Mathematical Society, vol. 5 (1930), pp. 148–149). The authors have found the first fourteen terms in the desired expansion. This makes it possible to compute q in terms of ϵ with an accuracy varying between seven places for $\epsilon = 0.4$ to eighteen places or better for $\epsilon \leqq 0.25$. These results were obtained in the course of work by the Mathematical Tables Project, Work Projects Administration for the City of New York, conducted under the sponsorship of the National Bureau of Standards. (Received December 17, 1941.)

115. A. N. Lowan and Abraham Hillman: *A short table of the zeros of the equation* $f(x) = J_0(x)\,Y_0(kx) - J_0(kx)\,Y_0(x) = 0$.

The Mathematical Tables Project conducted by the Work Projects Administration of New York City, under the sponsorship of the National Bureau of Standards, has computed a short table of the first five zeros of the above equation for $k = 1$ (0.5) 4.0. The zeros were first computed with the aid of the method of McMahon, Annals of Mathematics, vol. 9 (1894–1895), pp. 23–30. Each zero was recomputed by inverse

interpolation from the values of $f(x)$ for about six arguments at intervals of 0.01 in the neighborhood of the zero in question. As was anticipated from the fact that McMahon's formulae are based on the asymptotic expansions of Bessel functions, the accuracy of the zeros computed by the latter method increases with the order of the zeros. To illustrate, for $k=3$, the first zero is correct to only two decimal places whereas the fifth zero is correct to six places. When each one of the zeros has been computed for a number of k's at sufficiently small intervals, it is expected that the zero in question for any value of k within the given range will be easily obtainable by interpolation. (Received December 6, 1941.)

116. K. L. Nielsen: *Some properties of functions satisfying partial differential equations of elliptic type.*

The author considers the totality, T, of particular solutions of a partial differential equation, $L(u) \equiv \Delta u + a_1 \partial u/\partial x + a_2 \partial u/\partial y + a_3 u = 0$, $a_k = a_k(x, y)$, $[k=1, 2, 3]$. Using results of operators transforming analytic functions into solutions of $L(u)=0$; [see Bergman, Comptes Rendus de l'Academie des Sciences, Paris, vol. 205 (1937), p. 1360 and Matematicheskii Sbornik, vol. 44 (1937), p. 1169], it is shown that for certain types of L there is a subclass S of functions belonging to T with the property that there exists a denumerable set $u_n(x, y) \in S$ each of which satisfies an ordinary differential equation, $\sum_{r=1}^{k} A_r(x, y; n) d^k u(x, y)/dx^k$ (A_r being algebraic functions of x and y and depending on n in a simple way), and an analogous equation with respect to y. Every $u \in S$, regular in $x^2+y^2 \leq \rho^2$, can be expanded in the uniformly convergent series $\sum \alpha_n u_n(x, y)$ in this circle. The singularities of $u \in S$ may be branch points of the type that u can be decomposed into $u=u_1+u_2$, where u_2 is regular at the singular point and u_1 satisfies an ordinary differential equation analogous to the one above. If the function element $u=\sum A_{mn} x^m y^n$, $u \in S$, is given, the author indicates a procedure to determine from A_{mn} whether u has only singularities described above, and for the determination of the distances of these branch points from the origin. (Received January 29, 1942.)

117. Harry Pollard: *The generalized Stieltjes transform.*

In this paper the author studies the extension to the generalized Stieltjes transform (1) $f(x)=\int_0^\infty (x+t)^{-\rho} d\alpha(t)$ of the theory developed by Widder for the case $\rho=1$. Specifically, the following results are obtained: (i) It is established that $f(x)$ is also a Laplace-Stieltjes transform for positive x if and only if $\alpha(t)=o(t^\rho)$ as $t \to \infty$. (ii) The inversion of (1) is accomplished by means of a linear differential operator. (iii) Necessary and sufficient conditions are obtained for the representation of a function $f(x)$ in the most general form (1). (iv) Conditions are obtained for the representation of $f(x)$ in the form (1) with $\alpha(t)$ of preassigned type. The results (i) and (iii) are new even for $\rho=1$. (Received January 30, 1942.)

118. Raphaël Salem: *On singular monotonic functions of the Cantor type.*

The first part of the paper gives the construction of a singular monotonic function *of the Cantor type* with Fourier-Stieltjes coefficients of order $n^{(-1/2)+\epsilon}$ ($\epsilon>0$ as small as desired) and even $n^{-1/2}\Omega(n)$, $\Omega(n)$ increasing to infinity as slowly as desired. The second part gives some new examples of sets of uniqueness and sets of multiplicity for trigonometrical series and shows that a very simple mapping can transform a set of uniqueness into a set of multiplicity. The third part gives the construction of a con-

tinuous monotonic function $F(x)$ of the Cantor type such that $\int_0^{2\pi} \exp(n_p ix)\,dF$ tends to $F(2\pi)-F(0)$ for a sequence $\{n_p\}$ such that p/n_p tends to zero as slowly as desired. (Received January 27, 1942.)

119. A. C. Schaeffer: *On the oscillation of differential transforms.* III.

It is shown that if in an interval (a, b) all derivatives of a function exist and no derivative changes sign more than a fixed bounded number of times in the interval then the function is analytic in the interval. This answers a question which was raised by Pólya as a generalization of a theorem of S. Bernstein. In the case in which all derivatives of a function $f(x)$ exist for $-\infty<x<\infty$ and the function is of exponential growth, let $g(n)$ be the maximum number of variations in sign of $f^{(n)}(x)$ in any interval of length, say, 1. If $g(n)$ tends to infinity with n, but sufficiently slowly, it can be shown that $f(x)$ is an entire function of order 1. These results are obtained by first showing that there is a function $p(n)$ such that the following statement is true: if $f(x)$ is bounded by 1 in $(-1, 1)$ and its first n derivatives are continuous in this interval, then the inequality $|f'(0)|>p(n)$ implies that $f^{(n)}(x)$ changes sign at least $n-1$ times in the interval. The preceding papers in this series have been written by G. Szegö and Einar Hille. (Received January 28, 1942.)

120. L. L. Silverman and Otto Szász: *On a class of Nörlund matrices.*

The definition of a Nörlund matrix depends upon a sequence of numbers p_n. The Nörlund matrix is then a triangular matrix, whose elements are the numbers p_{n-k} divided by the sum of the numbers p_n from zero to n. A Nörlund matrix is defined to be of *finite rank* if $p_n\neq 0$ for some value of n, and $p_n=0$ for all greater n. A matrix of finite rank is *simple* if the numbers p_n are all zero or unity. In this paper some general properties of matrices of finite rank, and of simple matrices are obtained. A simple matrix for which *all* the numbers $p_n=1$, when $n<r$, and for which $p_n=0$ when $n\geqq r$ is called a Z-matrix. It is denoted by Z_r. The relative inclusion of corresponding summability methods among themselves and with the arithmetic mean methods is investigated. Among the results obtained are the following: if h is a factor of k then Z_h is included in Z_k; if h is prime to k, then the only sequences evaluated by both definitions are the convergent sequences. A study is also made of the inverse if the transformation which is a linear combination of the identity and Z_k. (Received January 22, 1942.)

121. Wolfgang Wasow: *On boundary layer problems in the theory of ordinary differential equations.*

Given a differential equation involving a parameter ρ in such a way that when ρ tends to infinity a "limiting" differential equation of lower order than the original one is obtained. What happens then to the solution $U(x, \rho)$ of a boundary value problem of the differential equation, if ρ tends to infinity? For ordinary linear differential equations of order n depending linearly on the parameter, and for a rather general class of boundary conditions not involving ρ, this question is answered in the paper by an easily applicable rule. This rule shows that in general $u(x)=\lim_{\rho\to\infty} U(x, \rho)$ exists only if the n boundary conditions are not too unevenly divided between the two end points. If the limit function exists it is a solution of the limiting differential equation but satisfies no longer all the boundary conditions prescribed for $U(x, \rho)$. The rule tells which of the n boundary conditions cease to be satisfied after the passage to the limit.

The proof consists in an asymptotic calculation of $U(x, \rho)$ based on the theory of asymptotic solution of ordinary linear differential equations involving a parameter, as developed by G. D. Birkhoff, Noaillon, Tamarkin, Trjitzinsky and others. (Received January 8, 1942.)

Applied Mathematics

122. Stefan Bergman: *Two-dimensional flow around two profiles.*

The study of the influence of tail surfaces on lift and pressure distribution of a wing can be reduced to the investigation of problems in the conformal mapping of doubly connected domains. The use of orthogonal functions enables one to give simple formulas for the lift and the moment in the case of a uniform flow around one and around two profiles. In the first case the lift, L, is found to be the expression $L=4(\pi)^{1/2}\rho V^2(\sum|\phi_\nu(0)|^2)^{1/2}\sin[\alpha+\pi-\arg(\sum{}^*\psi_\nu(b)\phi_\nu(0))]$, where ρ is the density, $Ve^{-i\alpha}$ the velocity at infinity, $\psi_\nu(z)=\int_0^z\phi_\nu(z)dz$, and $\{\phi_\nu(z)\}$ a complete system of orthonormal polynomials of a domain B. (B is the domain obtained from the exterior of the profile by the transformation $z=1/\zeta$, b is the coordinate of the cusp, and the summation $\sum^*$ is understood in a certain special sense.) Analogous formulas exist for the moment and similar ones in the case of a flow around two profiles. (See also Notes of Lectures on Conformal Mapping, publication of Brown University, chap. XI, §§5–7) (Received January 29, 1942.)

123. Henry Wallman: *On the reduction in harmonic distortion due to high frequency pre-emphasis.* Preliminary report.

It is now common practice in high-fidelity sound broadcasting, in either FM or AM, to employ high frequency pre-emphasis, the object being an increase in signal-to-noise ratio. An additional effect, namely a reduction in harmonic distortion, has been noted experimentally. An analysis of this reduction in distortion is made in this paper, and quantitative evaluations are given for single-tone harmonic distortion of all orders. (Received December 30, 1941.)

124. Alexander Weinstein: *Spherical pendulum and complex integration.*

The following theorem, due to Puiseux (Journal de Mathématiques, 1842) is proved by a simple application of the theory of residues: The increment of the azimuth of a spherical pendulum corresponding to its passage from the lowest level z_1 to the highest level z_2 is greater than $\pi/2$. The boundary of the domain in the complex z-plane to which Cauchy's theorem is applied consists of a cut connecting z_1 with z_2 and of a vertical straight line to the right of z_2. (Received January 26, 1942.)

Geometry

125. P. O. Bell: *The parametric osculating quadrics of a family of curves on a surface.*

In this paper the author investigates the properties of the parametric osculating quadrics of a family of curves on a surface. These quadrics were introduced by Dan Sun (Tôhoku Mathematical Journal, vol. 32 (1930), pp. 81–85). His definition is essentially the following: At three neighboring points P, P_1, P_2 on an asymptotic curve C_u of a surface S construct the tangents to the curves of a one-parameter family on S.

The three tangents determine a quadric whose limit, as P_1, P_2 independently approach P along C_u, is the parametric osculating quadric $Q^{(u)}$ of the one-parameter family of curves of S at P. A quadric $Q^{(v)}$ is similarly defined with respect to the other asymptotic curve C_v. The union curves of an arbitrary congruence Γ', the curves of Darboux and the curves of Segre are all characterized geometrically in association with the quadrics $Q^{(u)}$ and $Q^{(v)}$. This paper will appear in the American Journal of Mathematics. (Received January 29, 1942.)

126. Nathaniel Coburn: *Conformal geometry of unitary space.*

The principal purpose of this paper is the determination of those affinors of a unitary K_n which: (I) transform as affinors under the analytic group of K_n; (II) are invariant under conformal transformations of the fundamental tensor of K_n. An equi-conformal fundamental tensor and connection are introduced, both of which are invariant under (II). It is shown that if the group of K_n is equi-analytic (determinants of (I) are constant), then the components of the equi-conformal connection transform as do the components of the ordinary connection of K_n. By use of the equi-conformal connection, a conformal connection is determined which transforms as does the ordinary connection of K_n under (I) and which is invariant under (II). Necessary and sufficient conditions are given for the existence of $k \leqq n^2$ conformal fundamental tensors. Next, expressions for the conformal curvature affinor in terms of the ordinary connection of K_n are derived. It is shown that: (1) if the unitary space K_n is conformal unitary euclidean, then the conformal curvature affinor vanishes; (2) no unitary space of constant nonvanishing curvature $K_n (n>2)$ is conformal to a unitary euclidean space. (Received January 6, 1942.)

127. Nathaniel Coburn: *Congruences in unitary space.*

The general properties of the congruence affinors of ∞^{n-1} curves, which are imbedded in a unitary space of n dimensions, K_n, are developed. The case in which the ∞^{n-1} congruence curves are either real curves X_1 or unitary curves U_1 is completely characterized. Next, by a study of two systems of Pfaffians, two types of orthogonality are defined: (1) ∞^1 hypersurfaces which are *completely unitary* orthogonal to the congruence curves; (2) ∞^1 hypersurfaces which are *semi-unitary* orthogonal to the congruence curves. It is shown that: (1) the ∞^1 completely unitary orthogonal hypersurfaces are ∞^1 unitary K_{n-1}; (2) the ∞^1 semi-unitary orthogonal hypersurfaces are ∞^1 semi-analytic spaces X_{n-1}. An additional analytical characterization of these two types of hypersurfaces is given. The final section of the paper is devoted to two problems: (1) the characterization in terms of congruence affinors of these two types of hypersurfaces; (2) special properties of congruences associated with each type of unitary orthogonal hypersurface. The desired characterizations are obtained in each case. Further, special properties of these congruences are found. Many of these properties are extensions of congruence theorems in Riemannian space. (Received January 6, 1942.)

128. J. J. DeCicco: *A generalization of the dual-isothermal transformations.*

In this paper is discussed a generalization in the plane of the theorem that the only lineal element transformations which carry every dual-isothermal family of curves into a dual-isothermal family are $U=\phi$, $V=(a_2v+b_2w+c_2)/(a_1v+b_1w+c_1)$, $W=(a_3v+b_3w+c_3)/(a_1v+b_1w+c_1)$, where ϕ, a_k, b_k, c_k are functions of u only. A field

element may be defined by (u, v, w, p, q), where (u, v, w) are the hessian coordinates of the lineal elements, $p=w_u$, and $q=w_v$. All field element to lineal element transformations which carry every dual-isothermal family into a dual-isothermal family are $U=\phi$, $V=(a_2v+b_2p+c_2)/(a_1v+b_1p+c_1)$, $W=(a_3v+b_3p+c_3)/(a_1v+b_1p+c_1)$, where ϕ, a_k, b_k, c_k are functions of u, q, and $w-qv$ only. The contact field element transformations preserving the dual-isothermal character are also determined. (In the statement of this result, those transformations which carry every field into a single dual-isothermal field are excluded.) (Received December 4, 1941.)

129. J. J. DeCicco: *Point transformations by which straight lines correspond to circles.*

By means of some of Kasner's theorems (*The generalized Beltrami problem concerning geodesic representation*, Transactions of this Society, vol. 4 (1903), pp. 149–152), the author determines all the point transformations by which all the straight lines of the XY-plane correspond to circles of the xy-plane. These form the eleven-parameter set (not a group) $X=(a_2\{x^2+y^2\}+b_2x+c_2y+d_2)/(a_1\{x^2+y^2\}+b_1x+c_1y+d_1)$, $Y=(a_3\{x^2+y^2\}+b_3x+c_3y+d_3)/(a_1\{x^2+y^2\}+b_1x+c_1y+d_1)$. The family of circles consists of the ∞^2 circles orthogonal to a fixed circle (real or imaginary). Any transformation carrying more than $6\infty^1$ circles into straight lines must belong to our set. Conversely, any correspondence converting more than $6\infty^1$ straight lines into circles must be the inverse of a correspondence of our set. Obviously the set contains both the projective and the Moebius groups. (Received December 4, 1941.)

130. Jesse Douglas: *On the geodesic surfaces of a given curve family.*

Consider a family $\mathcal{F}$ of ∞^4 curves in space defined by differential equations of the form: $y''=F(x, y, z, y', z')$, $z''=G(x, y, z, y', z')$. Definitions: (1) relative to $\mathcal{F}$, a surface S is geodesic at one of its points p if every curve of $\mathcal{F}$ tangent to S at p lies entirely upon S; (2) a (totally) *geodesic surface* $\mathcal{G}$ is one which is geodesic at each of its points. In the general case, geodesic surfaces relative to $\mathcal{F}$ are non-existent. The following are shown to be the sole possibilities in which there is at least one geodesic surface through each curve C of $\mathcal{F}$: ∞^1 surfaces $\mathcal{G}$ through each curve C of $\mathcal{F}$ (*linear* family); exactly *two* surfaces $\mathcal{G}$ through each curve C (*intersectional* family, obtainable by cutting ∞^2 surfaces S with ∞^2 surfaces S'); exactly *one* surface $\mathcal{G}$ through each curve C (*semi-intersectional*). Examples of each of the preceding are easily given. Besides, the theory brings out the following possibilities: three or four surfaces $\mathcal{G}$ through each curve C, but the *effective* existence of these types seems doubtful. If n is a finite integer greater than 4, the type: n surfaces $\mathcal{G}$ through each curve C is definitely impossible. An important application is made to the author's work on the inverse problem of the calculus of variations: every curve family $\mathcal{F}$ of *intersectional* type is *extremal*, and in a highly general manner. (Received January 22, 1942.)

131. Edward Kasner and J. J. DeCicco: *A generalization of the isothermal transformations.*

In this paper is obtained a generalization of Kasner's theorem that the only lineal element transformations which carry every isothermal family into an isothermal family are $X+iY=F(x\pm iy)$, $\Theta=a\theta+h$, where a is a nonzero constant and h is a harmonic function. A field element (flat field) is defined by (x, y, θ, p, q), where (x, y) are the cartesian coordinates of the point, θ is the inclination, $p=\theta_x$, and $q=\theta_y$. There are seven types of field element to lineal element transformations which preserve the iso-

thermal character. In the real domain, there are three types: (I) and (II) $X+iY=F(x\pm iy,\ p\mp iq)$, $\Theta=a\theta+h$, where h is a biharmonic function; and (III) $X=\phi$, $Y=\psi$, $\Theta=h(X,\ Y)$, where ϕ and ψ are arbitrary functions, and h is a harmonic function of $(X,\ Y)$. Within the group of contact field element transformations, there are seven types. In the real domain there are five types: (I) and (II) The Kasner extended group; (III) and (IV) $p-iq=F(X\pm iY,\ x+iy)$, $\Theta=a[\theta-2\int(pdx+qdy)]+h(X,\ Y)$; and (V) $X=\phi$, $Y=\psi$, $\Theta=h(X,\ Y)$. Special cases are that the isogonal, the multiplicative, and the isocline trajectories of an isothermal family are always isothermal. (Received December 4, 1941.)

132. Edward Kasner and Don Mittleman: *A general theorem on the initial curvatures of dynamical trajectories.*

The theorem proved is an extension of Kasner's dynamical theorem which states that: If a particle starts from rest in any positional field of force, the initial curvature of the trajectory is one-third of the curvature of the line of force through the initial position. The generalized result is: If a particle starts from maximum rest in an acceleration field of order n, the initial curvature of the trajectory is $n!(n-1)!/(2n-1)!$ of the curvature of the line of force through the initial position. This paper will appear in the Proceedings of the National Academy of Sciences. (Received January 19, 1942.)

133. W. H. Roever: *Geometric statement of a fundamental theorem for four-dimensional orthographic axonometry.*

The theorem stated below makes possible the construction of three-dimensional models for the "picturization" of four-dimensional space in a manner analogous to that furnished by three-dimensional orthographic axonometry for the picturization of three-dimensional space on the plane. Theorem: Three conjugate diameters and the axis of revolution of an oblate spheroid may be regarded, as far as directions are concerned, as the orthographic projection on the three-dimensional space, in which the spheroid lies, of four mutually perpendicular concurrent axes of four-dimensional space. If these axes be taken as rectangular cartesian axes and the three-dimensional space of the spheroid as the picture space, then the ratios which the halves of the chosen conjugate diameters and the radius of the spheroid's focal circle bear to the spheroid's equatorial radius are the foreshortening ratios, that is, the numbers by which the scales on the axes of the four-dimensional space must be multiplied respectively in order to obtain those on the axonometric axes (that is, the three conjugate diameters and the axis of revolution of the given spheroid). (Received December 8, 1941.)

Logic and Foundations

134. A. R. Schweitzer: *On a class of ordered $(n+1)$-ads relevant to the algebra of logic.* I.

Within the frame of his geometric theory (American Journal of Mathematics, vol. 31, (1909), pp. 365–410, chap. 2, 3) the author generates complete classes of ordered $(n+1)$-ads $(n=1, 2, 3, \cdots)$ of generalized type of constituents in the algebra of logic by means of two equivalent processes: (1) Relatively to the given ordered $(n+1)$-ad $\alpha_1\alpha_2\cdots\alpha_{n+1}$ and the ordered dyads $(\alpha_1\lambda_1), \cdots, (\alpha_{n+1}\lambda_{n+1})$ the α's are replaced by the corresponding λ's either singly or in combination yielding $2^{n+1}(n+1)$-ads. (2) Relatively to α_1, λ_1 the elements $\alpha_2\alpha_3\cdots\alpha_{n+1}$; $\lambda_2\lambda_3\cdots\lambda_{n+1}$ are successively

adjoined at the right (left) yielding the complete sets $(\alpha_1\alpha_2;\ \lambda_1\alpha_2;\ \alpha_1\lambda_2,\ \lambda_1\lambda_2)$, and so on. It is then assumed that the preceding $(n+1)$-ads belong to the general abstract space $S_{n+1}(G, H)$ as defined by the author (abstract 46-9-438). When $G=H=$alternating (symmetric) group on $n+1$ variables, the α_i and λ_i are represented graphically as opposite vertices of regular polyhedra in euclidean $(n+1)$-space or opposite faces of duals of the latter. When $n=2$ the polyhedra are the octahedron and the cube. (Received January 30, 1942.)

135. A. R. Schweitzer: *On a class of ordered $(n+1)$-ads relevant to the algebra of logic.* II.

The author develops a finite algebra of logic in which the complete set of ordered $(n+1)$-ads of generalized constituent type relative to $\alpha_1\alpha_2\cdots\alpha_{n+1}$ and the ordered dyads $(\alpha_1\ \lambda_1),\cdots,(\alpha_{n+1}\lambda_{n+1})$ is expressed as a reflexive formal sum: $\sum(\alpha_1\alpha_2\cdots\alpha_{n+1})=\sum(I)$. The corresponding terms are $\sum(A_1\alpha_2\cdots\alpha_{n+1})$, $\sum(\alpha_1A_2\cdots\alpha_{n+1}),\cdots,\sum(A_1\ \alpha_2\cdots\alpha_{n+1}),\cdots,\sum(\alpha_1\alpha_2\cdots A_{n+1})$, where $\sum(A_1\alpha_2\cdots\alpha_{n+1})$ is the sum of all $(n+1)$-ads containing α_1, and so on. Then for $n=2$, for example, $\alpha_1\alpha_2\alpha_3=\sum(A_1A_2A_3)=\sum(A_1\alpha_2\alpha_3)\times(\alpha_1A_2\alpha_3)\times\sum(\alpha_1\alpha_2A_3)$, and so on. Also, $\sum(A_1\alpha_2\alpha_3)+\sum(A_1\alpha_2\alpha_3)=\sum(\alpha_1\alpha_2\alpha_3)$, and so on. Finally it is assumed that the preceding $(n+1)$-ads belong to the abstract relational space $S_{n+1}(G, G)$, where G is an arbitrary substitution group on $n+1$ variables (including the identical group). When G is the symmetric group the symbol $\sum(A_1\alpha_2\cdots\alpha_{n+1})$ can be replaced by the more economical symbol $\sum(A_1)$, and so on, and this case reduces essentially to a previous development by the author (abstract 47-9-430). (Received January 30, 1942.)

Statistics and Probability

136. Irving Kaplansky: *Note on a common error concerning kurtosis.*

In many text books there is to be found a statement to the effect that a frequency curve with positive (negative) kurtosis falls above (below) the corresponding normal curve in the neighborhood of the mean. In this note examples are given to show that there is actually no such connection between kurtosis and the height of the curve at its mean. (Received January 19, 1942.)

Topology

137. Paul Civin: *Two-to-one mappings of three-dimensional sets.*

This paper is concerned with the proof of the non-existence of a continuous mapping defined on the closed three-cell in which each inverse image consists of exactly two points. Corresponding theorems for the arc and two-cell were proved by O. G. Harrold (*The non-existence of a certain type of continuous transformation*, Duke Mathematical Journal, vol. 5 (1939), pp. 789–793) and J. H. Roberts (*Two-to-one transformations*, Duke Mathematical Journal, vol. 6 (1940), pp. 256–262), respectively. (Received January 26, 1942.)

138. F. B. Jones: *A certain non-metric Moore space.*

The purpose of this paper is to give an example of a non-metric Moore space which is nevertheless the sum of a monotone increasing collection of completely separable

domains each lying together with its boundary in the next. (Received December 31, 1941.)

139. J. L. Kelley: *Symmetric product spaces.* Preliminary report.

Let X be a metric separable arcwise connected space. If every pair (x, y) and (y, x) of $X \times X$ is identified, the resulting space is the symmetric product space X_2. Denote by X^* the subset of all points (x, x) of X_2. Then (a) the fundamental group of X_2 is always abelian; (b) if X is a polyhedron a necessary and sufficient condition that X be contractible is that X^* be a retract of X_2. (Received December 29, 1941.)

140. A. D. Wallace: *Chains and the structure of continua.*

The author gives a decomposition of compact (=bicompact) connected H-spaces into chains and p-chains analogous to that given by Kelley and Moore for separable spaces. It is shown that (i) if p is neither an end point nor a cut point then there is a point conjugate to p, (ii) hence each such point is contained in a unique p-chain, (iii) the meet of p-chains is null or a cut point, (iv) a chain is characterized as a continuum which contains each p-chain meeting it in at least two points, (v) each p-chain is the meet of a collection $|A|$ of continua such that $\overline{S-A}$ contains a finite set of components each containing one point of the p-chain. Examples show that chains and J-sets (Kelley) are distinct; p-chains and simple links and F-sets are equivalent in metric spaces and in general a simple link is a p-chain. (Received December 5, 1941.)

NEW PUBLICATIONS

BENNETT, K. B. See PEARSON, F. A.

BENTLEY, A. F. Linguistic Analysis of Mathematics. Bloomington, Indiana, Principia Press, 1941. 12+315 pp. $3.00.

CURTISS, D. R., edited by. Mathematical Monographs. Vol. 1. (Northwestern University Studies in Mathematics and the Physical Sciences, no. 1.) Evanston and Chicago. Illinois, Northwestern University, 1941. 7+172 pp. $2.25.

GREEN, S. L. Algebraic Solid Geometry. An Introduction. Cambridge University Press; New York, Macmillan, 1941. 133 pp. $1.75.

HUREWICZ, W., and WALLMAN, H. Dimension Theory. (Princeton Mathematical Series, vol. 4.) Princeton University Press; London, Humphrey Milford and Oxford University Press, 1941. 7+165 pp. $3.00.

KEITH, A. See MCARTHUR, N.

MCARTHUR, N., and KEITH A. Intermediate Algebra. London, Methuen, 1942. 10 +356 pp. 8s. 6d.

MERRIMAN, G. M. To Discover Mathematics. New York, Wiley; London, Chapman and Hall, 1942. 12+435 pp. $3.00.

PEARSON, F. A., and BENNETT, K. B. Statistical Methods Applied to Agricultural Economics. New York, Wiley, 1942. $4.00.

SCHILPP, P. A., edited by. The Philosophy of Alfred North Whitehead. (The Library of Living Philosophers, vol. 3.) Evanston and Chicago, Illinois, Northwestern University, 1941. 20+745 pp. $4.00.

Table of Natural Logarithms. Vol. 3. Contains the Logarithms of the Decimal Numbers from 0.0001 to 5.0000. New York, Work Projects Administration, 1941. 18+501 pp. $2.00.

WALLMAN, H. See HUREWICZ, W.

WIDDER, D. V. The Laplace Transform. Princeton University Press; London, Humphrey Milford and Oxford University Press, 1941. 10+406 pp. $6.00